Tnpsc Maths[Eng]

Portions From

6th to 10th Samacheer Books

Edited By

Winmeen Team

Samacheer Kalvi Maths

6th Std

1]

The smallest among the common multiples of two numbers is called their least common multiple (LCM).

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Common Multiple method

Step 1: List the multiples of the given numbers.

Step 2: Circle and write the common multiples

Step 3: The smallest common multiple is the LCM.

Given numbers: 16, 24

Multiples of 16: =16, 32, 48, 64, 80, 96, 112, 128, 144, 160,.....

Multiples of 24: = 24, 48, 72, 96, 120, 144, 168,.....

Common multiples of 16 and 24 = 48, 96, 144, ...

(The smallest multiple among the common multiples is the LCM)

∴ The LCM of 16 and 24 = 48.

Factorization method

Step1: Find the prime factors of the given numbers.

Step2: Circle the common prime factors

Step3: Find the product of the common factors. Multiply this product with independent factors.

Given numbers: 16, 24

Factors of 16

		Remainder
2	16	
2	8	-0
2	4	-0
2	2	-0
	1	-0

Factors of 24

		Remainder
2	24	
2	12	-0
2	6	-0
3	3	-0
	1	-0

Factors of 16 = 2 x 2 x 2 x 2

Factors of 24 = 2 x 2 x 2 x 3

LCM is the product of the common factors and independent factors.

LCM of 16 &24 = 2 x 2 x 2 x 2 x 3 = 48

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2.5.2 Greatest Common Divisor (G.C.D.)

We know that different numbers have common divisors. Among the common divisors the greatest divisor is the G.C.D.

There are 2 methods to find the G.C.D. of the given numbers.

Common divisor method

Step 1: Find the divisors of the given numbers.

Step 2: Circle and write the common divisors

Step 3: Among the common divisors the greatest divisor is the G.C.D.

Given numbers: 30, 42

Divisors of 30 : 1, 2, 3, 5, 6, 10, 15, 30

Divisors of 42 : 1, 2, 3, 6, 7, 14, 21, 42

Common divisors : 1, 2, 3, 6

G.C.D. = 6

Given numbers = 35, 45, 60

Divisors of 35 : 1, 5, 7, 35

Divisors of 45 : 1, 3, 5, 9, 15, 45

Divisors of 60 : 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Common divisors : 1, 5

G.C.D. : 5

Factorization method

Step 1: Find the prime factors of the given numbers.

Step 2: Circle the common prime factors.

Step 3: Product of the common factors is the G.C.D. of the given numbers.

Given numbers: 30, 42

Prime factors of 30

		Remainder
2	30	
3	15	– 0
5	5	– 0
	1	– 0

Prime factors of 42

		Remainder
2	42	
3	21	–0
7	7	–0
	1	–0

Factors of 30 = 2 x 3 x 5

Factors of 42 = 2 x 3 x 7

(circle the common factors)

G.C.D = 2 x 3 = 6

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Example : 3

Find the G.C.D. of 85, 45, 60 by factorization method.

Factors of 85	Factors of 45	Factors of 60
5 | 85 Remainder 17 | 17 -0 | 1 -0	3 | 45 Remainder 3 | 15 -0 5 | 5 -0 | 1 -0	2 | 60 Remainder 2 | 30 -0 3 | 15 -0 5 | 5 -0 | 1 -0

Factors of 85 = 5 x 17

Factors of 45 = 3 x 3 x 5

Factors of 60 = 2 x 2 x 3 x 5

(Circle the common factors of all the three numbers)

G.C.D. of the given numbers = 5.

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Example : 5

Find the capacity of the biggest vessel that can be used a whole number of times to empty three cans of capacity 36 litres, 48 litres and 60 litres respectively.

In this case we need to find the G.C.D of the three numbers 36, 48 and 60 so that a vessel of that capacity can empty all the three cans when used a whole number of times.

Prime factors of 36

		Remainder
2	36	
3	18	- 0
2	6	- 0
3	3	- 0
	1	- 0

Prime factors of 48

		Remainder
2	48	
3	24	- 0
2	8	- 0
2	4	- 0
2	2	- 0
	1	- 0

Prime factors of 60

		Remainder
2	60	
3	30	- 0
2	10	- 0
5	5	- 0
	1	- 0

Factors of 36 = 2 x 3 x 2 x 3

Factors of 48 = 2 x 3 x 2 x 2 x 2

Factors of 60 = 2 x 3 x 2 x 5 (Circle the common factors)

∴ G.C.D of 36, 48 and 60 = 2 x 3 x 2 = 12

∴ A vessel of 12 litre capacity can be used 3 times, 4 times and 5 times respectively to empty the three cans.

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Example : 6

Three medical representatives meet a doctor on a particular day. The first representative, the second representative and the third representative meet the doctor regularly on every 10^{th} day, 15^{th} day and 20^{th} day respectively. On which day all the representatives meet the doctor?

To find out the day on which all the three of them meet the doctor together is the LCM of these numbers.

Multiples of 10 :10,20,30,40,50,60,70,80,90,100,110,120 …

Multiples of 15 :15,30,45,60,75,90,105,120…

Multiples of 20 :20,40,60,80,100,120…

Common Multiples of 10,15,20 = 60,120…

L.C.M of 10,15 and 20 = 60

∴ Three of them meet the doctor on the 60th day.

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Product of two numbers = G.C.D. x L.C.M.

Example : 7

The G.C.D. of 36, 156 is 12.
Find their L.C.M.

First number = 36

Second number = 156

G.C.D. = 12

$$\text{L.C.M.} = \frac{\text{Product of the two numbers}}{\text{G.C.D.}}$$

$$= \frac{36 \times 156}{12}$$

$$= 468$$

Example : 8

The G.C.D. and L.C.M. of two numbers are 3 and 72 respectively. If one number is 24. Find the other.

One number = 24

G.C.D. = 3

L.C.M. = 72

$$\text{Other number} = \frac{\text{G.C.D. x L.C.M.}}{\text{One number}}$$

$$= \frac{3 \times 72}{24}$$

$$= 9$$

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Example : 8

$$\frac{3}{8} + \frac{5}{7} = \frac{(3x7)+(5x8)}{8x7}$$

$$= \frac{21+40}{56}$$

$$= \frac{61}{56}$$

Example : 9

$$\frac{11}{10} + \frac{4}{9} = \frac{(11x9)+(4x10)}{10 \times 9}$$

$$= \frac{99+40}{90}$$

$$= \frac{139}{90}$$

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Note: Mixed fraction = Natural number + proper fraction.

$4\frac{1}{2} = 4 + \frac{1}{2}$ and $22\frac{1}{3} = 22 + \frac{1}{3}$

3.1.8 Conversion of improper fractions into mixed fractions.

Example : 10

$$\frac{7}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3}$$

$$= \frac{6}{3} + \frac{1}{3}$$

$$= 2 + \frac{1}{3} = 2\frac{1}{3}$$

Divide 7 by 3

$$\begin{array}{r} 2 \\ 3\overline{)\,7} \\ 6 \\ \hline 1 \\ \hline \end{array}$$

Divisor = 3
Quotient = 2
Remainder = 1

$$\boxed{\text{Mixed fraction} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}}$$

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3.1.9: Conversion of mixed fractions into improper fractio

Example : 11

Convert $3\frac{2}{7}$ into improper fraction.

$$3\frac{2}{7} = 3+\frac{2}{7} = 1+1+1+\frac{2}{7}$$
$$= \frac{7}{7}+\frac{7}{7}+\frac{7}{7}+\frac{2}{7}$$
$$= \frac{7+7+7+2}{7} = \frac{23}{7} \qquad \boxed{3\frac{2}{7} = \frac{23}{7}}$$

$$\text{Improper fraction} = \frac{(\text{Natural number x denominator}) + \text{Numerator}}{\text{Denominator}}$$

$$3\frac{2}{7} = \frac{(3\times 7)+2}{7}$$
$$= \frac{21+2}{7} = \frac{23}{7}$$

∴ The improper fraction of $3\frac{2}{7}$ is $\frac{23}{7}$

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Activity

Answer the following :

(i) 4 times $\frac{1}{4}$ (ii) one – fourth of 12 (iii) 5 times $\frac{1}{5}$ (iv) Half of Half.

Example : 14

$\frac{1}{3}$ x 9 = ?

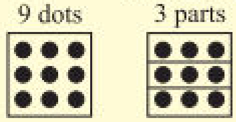
$\frac{1}{3}$ of 9 = 3

(or)

$\frac{1}{3}$ x 9 = $\frac{9}{3}$ = 3

9 dots

Divided into 3 parts



Example : 15

Find the value : $\frac{2}{3}$ of 12

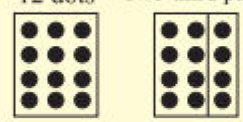
$\frac{2}{3}$ of 12 = 8

(or)

$\frac{2}{3}$ x 12 = $\frac{24}{3}$ = 8

12 dots

Divided into Two-third parts



Activity

Answer the following :

(i) $\frac{3}{4}$ of 16 (ii) $\frac{2}{14}$ x 28 (iii) $\frac{1}{4}$ of 16 (iv) $\frac{5}{15}$ x 45

Example : 16

There are 20 balls in a box. How many balls should be taken from the box, if you want to take three quarters of them?

Solution:

Total No.of balls = 20

Balls to be taken = $\frac{3}{4} \times 20$

= 3 x 5

= 15 balls

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Example : 17

There are 60 students in a class. $\frac{2}{5}$ of them are boys. Find the number of boys

Solution

Total number of students = 60

No. of boys = $\frac{2}{5}$ x 60

= 2 x 12 = 24 boys

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Example : 22

Change the following fractions into decimal fractions.

(i) $30 + 8 + \frac{4}{10}$ (ii) $400 + 80 + \frac{6}{10}$

Solution:

(i) $30 + 8 + \frac{4}{10}$ (ii) $400 + 80 + \frac{6}{10}$

$= 38 + 0.4 = 38.4$ $= 480 + 0.6 = 480.6$

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Example : 21

Write the decimal numbers for the following:

1) Four ones and 3 tenths
2) Seventy two and 6 tenths.

Solution: i) Four ones and 3 tenths

$$4 + \frac{3}{10} = 4 + 0.3 = 4.3$$

ii) Seventy two and 6 tenths

$$72 + \frac{6}{10} = 72 + 0.6 = 72.6$$

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Example : 28

Convert into decimals (i) $\frac{4}{100}$ (ii) $\frac{36}{100}$ (iii) $6+\frac{7}{10}+\frac{8}{100}$

Solution

(i) $\frac{4}{100}=0.04$ (ii) $\frac{36}{100}=0.36$ (iii) $6+\frac{7}{10}+\frac{8}{100}=6+\frac{70}{100}+\frac{8}{100}$

$=6+\frac{78}{100}$

$=6+0.78=6.78$

Activity

Do it Yourself

Convert into decimal numbers

(i) $\frac{6}{100}$ (ii) $\frac{36}{100}$ (iii) $200+80+9+\frac{3}{100}$

Example : 29

Write the decimal number: Eighteen and forty five hundredths.

Solution

Eighteen and forty five hundredths $=18+\frac{45}{100}=18+0.45=18.45$

Example : 30

Convert the following decimal numbers into fractions

(i) 0.09 (ii) 0.83

Solution

(i) $0.09=\frac{9}{100}$ (ii) $0.83=\frac{83}{100}$

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1.2 Ratio

- Ratio is a way to compare two or more quantities of the same kind
- The ratio of two non-zero quantities a and b is written as a : b. It is read as "a is to b"
- The ratio is represented by the symbol " : "
- a and b are called as the terms of the ratio. 'a' is called as the antecedent and 'b' is called as the consequent
- The ratio is represented in numbers and it does not have any unit.
- Order in a ratio is important. a : b is different from b : a.

For example : there are 15 boys and 12 girls in a class.

The ratio of boys to girls is 15 : 12 while the ratio of girls to boys is 12: 15.

- When two quantities a and b are compared they must be in the same unit .

For example: If a = 1m 20 cm and b = 90 cm then a must be written as 120 cm and b= 90 cm

and the ratio a : b is 120 : 90

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Example : 2

S.No.	Quantity	Ratio form	Fraction form	Reduced form
1.	Ratio of 15 men and 10 women	15 : 10	$\frac{15}{10}$	3 : 2
2.	Ratio of 500 gm and 1 kg	500 : 1000	$\frac{500}{1000}$	1 : 2
3.	Ratio of 1 m 25 cm and 2m	125 : 200	$\frac{125}{200}$	5 : 8

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Example : 5

In a Village of 10,000 people, 4,000 are Government Employees and the remaining are self-employed. Find the ratio of

i) Government employees to people of the village.

ii) Self employed to people of the village.

iii) Government employees to self-employed.

Solution :

Number of people in the village = 10,000

Number of Government employees = 4,000

$\therefore$ Self employed = 10,000 – 4,000 = 6,000

S.No.	Quantity	Ratio form	Fraction form	Lowest form of the Ratio
1.	Government employees to people of the village.	4000 : 10000	$\frac{4000}{10000}$	2 : 5
2.	Self employed to people of the village.	6000 : 10000	$\frac{6000}{10000}$	3 : 5
3.	Government employees to self employed.	4000 : 6000	$\frac{4000}{6000}$	2 : 3

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Example : 6

Write any 5 equivalent ratios for 5 : 7

Solution :

Given ratio = 5 : 7

The ratio in fractional form = $\frac{5}{7}$

The equivalent fractions of $\frac{5}{7}$ are $\frac{10}{14}, \frac{15}{21}, \frac{20}{28}, \frac{25}{35}, \frac{55}{77}$

$\therefore$ The equivalent ratios of 5 : 7 are 10 : 14, 15 : 21, 20 : 28, 25 : 35 and 55 : 77

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Example : 7

Compare 3:5 and 4:7

We have to compare $\frac{3}{5}$ and $\frac{4}{7}$

The L.C.M of denominator 5 and 7 is 35.

$\frac{3}{5} = \frac{3}{5} \times \frac{7}{7} = \frac{21}{35}$ $\quad\quad$ $\frac{4}{7} = \frac{4}{7} \times \frac{5}{5} = \frac{20}{35}$

$\frac{21}{35}$ is greater than $\frac{20}{35}$

$\therefore \frac{3}{5}$ is greater than $\frac{4}{7}$

Hence 3:5 is greater than 4:7

Example : 8

Divide ₹. 280 in the ratio 3:5

3:5 means the first quantity is 3 parts and the second quantity in 5 parts.

The Total number of parts = 3 + 5 = 8

8 parts = ₹.280

$\therefore$ 1part = $\frac{280}{8}$ =35

$\therefore$3 parts = $3 \times 35 =$ Rs.105

and 5 parts =5×35 =₹.175

Parts	Amount
8	280
3	?
5	?

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The length and breadth of a rectangle are in the ratio 4:7. If the breadth is 77cm, find the length?

Breadth = 77cm

The ratio of length to breadth is 4:7

Parts	Measurements
7	77
1	?
4	?

Breadth = 7 parts

7parts = 77cm

1part = $\frac{77}{7}$ cm = 11cm

length = 4 parts

4parts = 4×11 cm = 44cm

$\therefore$ Length of the rectangle = 44cm.

Example : 10

In a village of 1,21,000 people, the ratio of men to women is 6 : 5

Find the number of men and women?

Solution : Number of people in the village = 1,21,000

Ratio of men to women = 6 : 5

Total number of parts = 6 + 5 = 11

Parts	No. of people
11	121000
6	?
5	?

11 parts = 1,21,000

$\therefore$ 1 part $= \frac{1,21,000}{11} = 11,000$

$\therefore$ Number of men in the village $= 6 \times 11,000 = 66,000$

$\therefore$ Number of women in the village $= 5 \times 11,000 = 55,000$

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1.5 Proportion

When two ratios expressed in its simplest form are equal they are said to be in proportion.

Proportion is represented by the symbol ‘ = ‘ or ‘:: ‘

If the ratio a: b is equal to the ratio c : d then a,b,c,d are said to be in proportion.

Using symbols we write as a: b = c: d or a: b :: c: d

Example : 11

1. Show that the ratios (i) 2 : 3, 8 : 12, (ii) 25 : 45, 35 : 63 are in proportion.

Solution :

	Ratio form	Fraction form	Simplified form
i)	2:3	$\frac{2}{3}$	2:3
	8:12	$\frac{8}{12}=\frac{2}{3}$	2:3
	$\therefore$ 2:3, 8:12 are in proportion		
ii)	25:45	$\frac{25}{45}=\frac{5}{9}$	5:9
	35:63	$\frac{35}{63}=\frac{5}{9}$	5:9
	$\therefore$ 25:45, 35:63 are in proportion		

Note : In the above example (ii), multiply 45 by 35 and 25 by 63

We get $25 \times 63 = 45 \times 35 = 1575$

If a : b and c : d are in proportion then $a \times d = b \times c$

The proportion is written as a : b :: c : d

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In a proportion, the product of extremes is equal to the product of means.

Example : 12

Show that 12 : 9, 4 : 3 are in proportion.

Solution : The product of the extremes $= 12 \times 3 = 36$

The product of the means $= 9 \times 4 = 36$

$\therefore$ 12 : 9, 4 : 3 are in proportion

(i.e.) 12 : 9 :: 4 : 3

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Example : 13

Find the missing term in 3 : 4 = 12 : ____

Solution :

The product of the extremes = The product of the means

Therefore 3 × ____ = 4 × 12 ; By dividing both sides by 3

we get the missing term = $\frac{4 \times 12}{3}$ = 16

Example : 14

Using 3 and 12 as means, write any two proportions.

Given 3 and 12 are means

So, __:3 = 12 : __

The product of the means 3 × 12 = 36

The product of Extremes must be 36

36 can be written as 2 × 18 or 4 × 9 etc,

∴ 2:3=12:18   4:3=12:9

Two proportions are 2:3::12:18 and 4:3::12:9

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Example : 15

If the cost of a book is ₹.12, find the ratio of 2, 5, 7 books to their cost.

What do you observe from this?

No. of books	Total Cost	Ratio	Fraction form	Simplified form
2	2 × 12 = 24	2 : 24	$\frac{2}{24}$	1 : 12
5	5 × 12 = 60	5 : 60	$\frac{5}{60}$	1 : 12
7	7 × 12 = 84	7 : 84	$\frac{7}{84}$	1 : 12

From the above table, we find that the ratio of the number of books to the cost of books are in proportion.

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1.6 Direct Variation

Two quantities are said to be in direct variation if an increase (or decrease) in one quantity results in increase (or decrease) in the other quantity. (i.e.) If two quantities vary always in the same ratio then they are in direct variation.

Example : 16

Shabhana takes 2 hours to travel 35 km. How much distance she will travel in 6 hours?

Solution : When time increases the distance also increases.

Therefore, they are in direct variation

Time (hrs)	Distance (km)
2	35
6	?

$2 : 6 = 35 : \square$

missing term $= \frac{6 \times 35}{2} = 105$

Shabana has travelled 105 km in 6 hou₹.

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Example : 17

The cost of uniforms for twelve students is ₹.3,000. How many students can get uniform for ₹.1250.

Solution :

No. of students	Cost of the uniform ₹.
12	3,000
?	1,250

When money spent decreases the number of uniform also drecreases.

They are in direct variation

$12 : \square = 3000 : 1250$

Missing Term $= \frac{12 \times 1250}{3000} = 5$

5 students can be given uniform for ₹.1,250.

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Example : 18

Verify whether the following represents direct variation.

Numbers of books	10	8	20	4
Cost (in ₹.)	25	20	50	10

Arrange the data in ascending order.

Numbers of books	4	8	10	20
Cost (in ₹.)	10	20	25	50

Here the ratios are $\frac{4}{10} = \frac{2}{5}, \frac{8}{20} = \frac{2}{5}, \frac{10}{25} = \frac{2}{5}, \frac{20}{50} = \frac{2}{5}$

$$\therefore \frac{4}{10} = \frac{8}{20} = \frac{10}{25} = \frac{20}{25}$$

Here all the ratios are equal.

$\therefore$ They are in direct variation.

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Example : 19

A map is drawn to the scale of 1cm to 200km.

(i) What is the representive fraction.

(ii) If the distance between Nellai and Chennai is 3cm on this map, what is the actual distance between the two places?

Note the drawn length and the actual length are not in the same unit.

Therefore convert them into the same unit.

Now 200 km = 200 × 100000 cm [$\because$ 1km = 100000cm]

= 2,00,00,000cm

(i) The representive fraction = $\frac{1}{20000000}$

(ii) The distance between Nellai and Chennai (on the map) = 3 cm

The actual distance between Nellai and Chennai = 3 × 200 = 600 km

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1 minute	= 60 seconds
1 hour	= 60 minutes = 60 × 60 seconds
	= 3600 seconds
1 day	= 24 hours = 1440 minutes (24 × 60)
	= 86,400 seconds (24 × 60 × 60)

60 seconds	= 1 minute
1 sec	$= \frac{1}{60}$ minute
60 minutes	= 1 hour
1 minute	$= \frac{1}{60}$ hour

Example : 1

Convert 120 Seconds into minutes

$\because$ 60 seconds = 1 minute

1 second $= \frac{1}{60}$ minute

Solution:

120 seconds $= 120 \times \frac{1}{60} = \frac{120}{60} = 2$ minutes

120 seconds = 2 minutes

Example : 2

Convert 360 minutes into hours

60 minutes = 1 hour

$\therefore$ 1 minute $= \frac{1}{60}$ hour

Solution :

360 minutes $= 360 \times \frac{1}{60} = 360/60 = 6$ hours

360 minutes = 6 hours.

Example : 3

Convert 3 hours 45 minutes into minutes

Solution : 1 hour = 60 minutes

3 hours = 3 × 60 = 180 minutes

3 hours and 45 minutes = 180 minutes + 45 minutes

= 225 minutes.

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Example : 4

Convert 5400 seconds into hours

3600 seconds = 1 hour

$\therefore$ 1 second $= \frac{1}{3600}$ hour

Solution :

5400 Seconds $= 5400 \times \frac{1}{3600}$ hour

$= \frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$ hours.

5400 seconds $= 1\frac{1}{2}$ hours.

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Example : 5

Convert 2 hours 30 minutes 15 seconds into seconds.

Solution : 1 hour = 3600 seconds $\Rightarrow$ 2 hours = $2 \times 3600 = 7200$ seconds

1 minute = 60 seconds $\Rightarrow$ 30 minutes = $30 \times 60 = 1800$ seconds

2 hours 3 minutes 15 seconds = 7200 + 1800 +15 = 9015 seconds.

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Example : 6

Convert the following into 24 hour format.

i) 8 a.m. ii) 12 noon iii) 5.30 p.m.

i) In this case when the time is before noon the time is same in the 12 hour and 24 hour format. $\therefore$ 8 a.m. = 8.00 hours

ii) 12 noon = 12 hours

iii) for time in the afternoon add 12 to the given time

$\therefore$ 5.30 pm will become 5.30 + 12 = 17.30 hours.

35]

Example : 7

Convert the following into 12 hour format

i) 6.00 hours ii) 23.10 hours iii) 24 hours

i) If the number is less than 12 it will be taken as am and the time remains same $\therefore$ 6.00 hours = 6 .00 a.m.

ii) If it is greater than 12, 12 will be subtracted from the given time and it will be taken as p.m.

23.10 – 12 = 11.10 p.m.

iii) 24 hours = 24 – 12 = 12 midnight

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Example : 8

Find the duration of time from 4.00 a.m. to 4.00 p.m.

Solution :

4.00 pm = 4 + 12 = 16 hours.

4.00 am = 4 hours

∴ Duration of time interval = 16 – 4 = 12 hours

Example : 9

Cheran Express departs from Chennai at 22.10 hours and reaches Salem at 02.50 hours the next day. Find the jouney time.

Solution :

Arrival at Salem	= 02.50 hrs.
Departure time form Chennai (previous day)	= 22.10 hrs.
Journey time	= (24.00 – 22.10) + 2.50 = 1.50 + 2.50 = 4.40

∴ Journey time = 4 hours 40 minutes.

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Example : 10

A boy went to school at 9.00 a.m. After school, he went to his friend's house and played. If he reached back home at 5.30 p.m. find the duration of time he spent out of his house.

Solution :

Starting time from home	= 9.00 a.m.
Duration between starting time and 12.00 noon	= 12.00 – 9.00
	= 3.00 hours
Reaching time (home)	= 5.30 p.m

∴ Duration of time he spent out of his house = 3.00 + 5.30 = 8.30 hours.

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1day	=	24 hours
1 week	=	7 days
1 year	=	12 months
1 year	=	365 days
1 leap year	=	366 days
10 years	=	1 decade
100 years	=	1 century
1000 years	=	1 millennium

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Usually we denote length as 'l', breadth as 'b'

$\therefore$ Area of a rectangle = $(l \times b)$ sq. units

Example : 7

Find the area of a rectangle whose length is 8 cm and breadth 5 cm

Area of a rectangle = length $\times$ breadth = 8 cm $\times$ 5 cm = 40 sq. cm

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Example : 8

Find the area of a square of side 7 cm.

Area of a square = side $\times$ side = 7cm $\times$ 7cm = 49 sq. cm.

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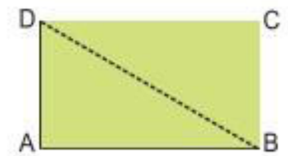
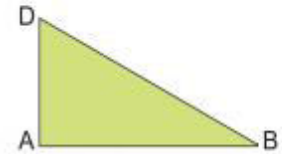
Area of a right triangle



Take a rectangular shaped card-board and cut it through a diagonal. We get 2 right triangles.



Area of a right triangle = half the area of the rectangle

$= \frac{1}{2} \times$ (length x breadth) sq. units

From this you know that
Area of a rectangle = Area of two right triangles.

The length and breadth of the rectangle become the base and height of the right triangle. Length is used as the base and breadth is used as the height.

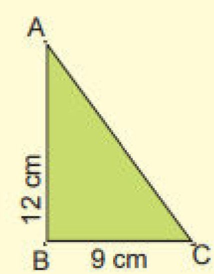
Hence, area of a right triangle $= \frac{1}{2} \times$ (base x height) sq.units.

> If base is denoted as 'b' and height as 'h', then the area of a right triangle $= \frac{1}{2}(b \times h)$ sq.units.

42]

Example : 9

Find the area of the following right triangle.



Solution :

Area $= \frac{1}{2} \times$ base $\times$ height

Base of triangle = 9 cm

Height = 12 cm

$\therefore$ Area $= \frac{1}{2} \times 9 \times 12 = 9 \times 6 = 54$ sq.cm.

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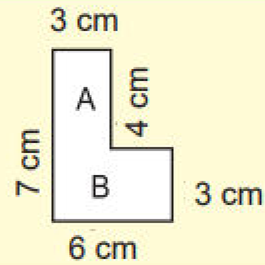
Example : 10

Find the area of the following shape.



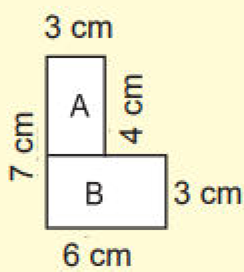
Solution : There are three methods to solve this problem.

I method

Area of (A) = $4 \times 3 = 12$ sq. cm.

Area of (B) = $6 \times 3 = 18$ sq. cm.

Therefore, area of the shape = 30 sq. cm.

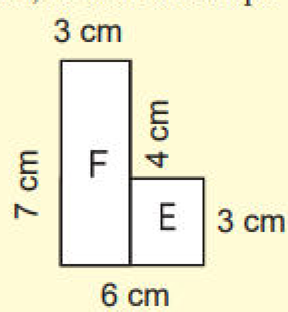


II method

Area of (F) = $7 \times 3 = 21$ sq. cm.

Area of (E) = $3 \times 3 = 9$ sq. cm.

Therefore, area of the shape = 30 sq. cm.



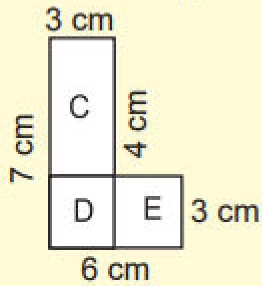
III method

Area of (C) = 4 × 3 = 12 sq. cm.

Area of (D) = 3 × 3 = 9 sq. cm.

Area of (E) = 3 × 3 = 9 sq. cm.

Therefore, area of the shape = 30 sq. cm.



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Points to remember

- The Perimeter of a closed figure is the total measure of the boundry.
- The Perimeter of a rectangle = 2 × (l + b) units.
- The Perimeter of a Square = (4 × s) units.
- The area of an object is the space occupied by it on a plane surface.
- The area of a rectangle = (l × b) sq. units
- The area of a Square = (s × s) sq.units.
- The area of a right angled triangle = $\frac{1}{2}$ × (base × height).
- The area of a shape do not change when they are rotated or move from their places.

7th Std

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Example (i)

Simplify: $\frac{2}{5}+\frac{3}{5}$

Solution

$$\frac{2}{5}+\frac{3}{5}=\frac{2+3}{5}=\frac{5}{5}=1$$

Example (ii)

Simplify: $\frac{2}{3}+\frac{5}{12}+\frac{7}{24}$

Solution

$$\frac{2}{3}+\frac{5}{12}+\frac{7}{24}=\frac{2\times8+5\times2+7\times1}{24}$$
$$=\frac{16+10+7}{24}$$
$$=\frac{33}{24}=1\frac{3}{8}$$

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Example (iii)

Simplify: $5\frac{1}{4} + 4\frac{3}{4} + 7\frac{5}{8}$

Solution

$$\begin{aligned} 5\frac{1}{4} + 4\frac{3}{4} + 7\frac{5}{8} &= \frac{21}{4} + \frac{19}{4} + \frac{61}{8} \\ &= \frac{42 + 38 + 61}{8} = \frac{141}{8} \\ &= 17\frac{5}{8} \end{aligned}$$

Example (iv)

Simplify: $\frac{5}{7} - \frac{2}{7}$

Solution

$$\frac{5}{7} - \frac{2}{7} = \frac{5-2}{7} = \frac{3}{7}.$$

Example (v)

Simplify: $2\frac{2}{3} - 3\frac{1}{6} + 6\frac{3}{4}$

Solution

$$\begin{aligned} 2\frac{2}{3} - 3\frac{1}{6} + 6\frac{3}{4} &= \frac{8}{3} - \frac{19}{6} + \frac{27}{4} \\ &= \frac{32 - 38 + 81}{12} \\ &= \frac{75}{12} = 6\ \frac{1}{4} \end{aligned}$$

47]

Example 1.9

Find : $\frac{1}{4}$ of $2\frac{1}{5}$

Solution

$$\begin{aligned} \frac{1}{4} \text{ of } 2\frac{1}{5} &= \frac{1}{4} \times 2\frac{1}{5} \\ &= \frac{1}{4} \times \frac{11}{5} \\ &= \frac{11}{20} \end{aligned}$$

48]

Example 1.10

In a group of 60 students $\frac{3}{10}$ of the total number of students like to study Science, $\frac{3}{5}$ of the total number like to study Social Science.

(i) How many students like to study Science?

(ii) How many students like to study Social Science?

Solution

Total number of students in the class = 60

(i) Out of 60 students, $\frac{3}{10}$ of the students like to study Science.

Thus, the number of students who like to study Science = $\frac{3}{10}$ of 60

$$= \frac{3}{10} \times 60 = 18.$$

(ii) Out of 60 students, $\frac{3}{5}$ of the students like to study Social Science.

Thus, the number of students who like to study Social Science

$$= \frac{3}{5} \text{ of } 60$$

$$= \frac{3}{5} \times 60 = 36.$$

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(iii) Multiplication of a fraction by a fraction

Example 1.11

Find $\frac{1}{5}$ of $\frac{3}{8}$.

Solution

$$\frac{1}{5} \text{ of } \frac{3}{8} = \frac{1}{5} \times \frac{3}{8} = \frac{3}{40}$$

Example 1.12

Find $\frac{2}{9} \times \frac{3}{2}$.

Solution

$$\frac{2}{9} \times \frac{3}{2} = \frac{1}{3}$$

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Example 1.13

Leela reads $\frac{1}{4}^{th}$ of a book in 1 hour. How much of the book will she read in $3\frac{1}{2}$ hours?

Solution

The part of the book read by leela in 1 hour $= \frac{1}{4}$

So, the part of the book read by her in $3\frac{1}{2}$ hour $= 3\frac{1}{2} \times \frac{1}{4}$

$$= \frac{7}{2} \times \frac{1}{4}$$

$$= \frac{7 \times 1}{4 \times 2}$$

$$= \frac{7}{8}$$

$\therefore$ Leela reads $\frac{7}{8}$ part of a book in $3\frac{1}{2}$ hours.

Try these

Find

i) $\frac{1}{3} \times \frac{7}{5}$

ii) $\frac{2}{3} \times \frac{8}{9}$

51]

Example 1.14

Find (i) $6 \div \frac{2}{5}$ (ii) $8 \div \frac{7}{9}$

Solution

(i) $6 \div \frac{2}{5} = 6 \times \frac{5}{2} = 15$

(ii) $8 \div \frac{7}{9} = 8 \times \frac{9}{7} = \frac{72}{7}$

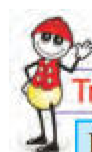
While dividing a whole number by a mixed fraction, first convert the mixed fraction into improper fraction and then solve it.

Example 1.15

Find $6 \div 3\frac{4}{5}$

Solution

$$6 \div 3\frac{4}{5} = 6 \div \frac{19}{5} = 6 \times \frac{5}{19} = \frac{30}{19} = 1\frac{11}{19}$$



Try these

Find:

i) $6 \div 5\frac{2}{3}$ ii) $9 \div 3\frac{3}{7}$

52]

Example 1.20

Add $\frac{9}{5}$ and $\frac{7}{5}$.

Solution

$$\frac{9}{5}+\frac{7}{5} = \frac{9+7}{5} = \frac{16}{5}.$$

Let us add two rational numbers with different denominators.

Example 1.21

Simplify: $\frac{7}{3}+\left(\frac{-5}{4}\right)$

Solution

$$\frac{7}{3}+\left(\frac{-5}{4}\right)$$

$$= \frac{28-15}{12} \quad \text{(L.C.M. of 3 and 4 is 12)}$$

$$= \frac{13}{12}$$

53]

Example 1.22

Simplify : $\frac{-3}{4}+\frac{1}{2}-\frac{5}{6}$.

Solution

$$\frac{-3}{4}+\frac{1}{2}-\frac{5}{6} = \frac{(-3\times 3)+(1\times 6)-(5\times 2)}{12} \text{ (L.C.M. of 4,2 and 6 is 12)}$$

$$= \frac{-9+6-10}{12}$$

$$= \frac{-19+6}{12} = \frac{-13}{12}$$

54]

(ii) Subtraction of rational numbers

Example 1.23

Subtract : $\frac{8}{7}$ from $\frac{10}{3}$

Solution:

$$\frac{10}{3} - \frac{8}{7} = \frac{70-24}{21} = \frac{46}{21}$$

Example 1.24

Simplify $\frac{6}{35} - \left(\frac{-10}{35}\right)$

Solution:

$$\frac{6}{35} - \left(\frac{-10}{35}\right) = \frac{6+10}{35} = \frac{16}{35}$$

55]

Example 1.25

Simplify : $\left(-2\frac{7}{35}\right) - \left(3\frac{6}{35}\right)$

Solution

$$\left(-2\frac{7}{35}\right) - \left(3\frac{6}{35}\right) = \frac{-77}{35} - \frac{111}{35}$$

$$= \frac{-77-111}{35} = \frac{-188}{35} = -5\frac{13}{35}$$

56]

Example 1.27

Find the product of $\left(\frac{4}{-11}\right)$ and $\left(\frac{-22}{8}\right)$.

Solution

$\left(\frac{4}{-11}\right)\times\left(\frac{-22}{8}\right)$

$=\left(\frac{-4}{11}\right)\times\left(\frac{-22}{8}\right) = \frac{88}{88}$

$= 1$

Example 1.28

Find the product of $\left(-2\frac{4}{15}\right)$ and $\left(-3\frac{2}{49}\right)$.

Solution

$$\left(-2\frac{4}{15}\right)\times\left(-3\frac{2}{49}\right) = \left(\frac{-34}{15}\right)\times\left(\frac{-149}{49}\right)$$

$$= \frac{5066}{735} = 6\frac{656}{735}$$

57]

Find $\left(\frac{2}{3}\right)\div\left(\frac{-5}{10}\right)$.

Solution

$$\left(\frac{2}{3}\right)\div\left(\frac{-5}{10}\right) = \frac{2}{3}\div\left(\frac{-1}{2}\right)$$

$$= \frac{2}{3}\times(-2) = \frac{-4}{3}$$

Example 1.31

Find $4\frac{3}{7}\div 2\frac{3}{8}$.

Solution

$$4\frac{3}{7}\div 2\frac{3}{8} = \frac{31}{7}\div\frac{19}{8}$$

$$= \frac{31}{7}\times\frac{8}{19} = \frac{248}{133}$$

$$= 1\frac{115}{133}$$

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Example 1.36

The side of a square is 3.2 cm. Find its perimeter.

Do you know?

Perimeter of a square = 4 × side

Solution

All the sides of a square are equal.

Length of each side = 3.2 cm.

Perimeter of a square = 4 × side

Thus, perimeter = 4 × 3.2 = 12.8 cm.

Example 1.37

The length of a rectangle is 6.3 cm and its breadth is 3.2 cm. What is the area of the rectangle?

Solution

Length of the rectangle = 6.3 cm

Breadth of the rectangle = 3.2 cm.

Area of the rectangle = (length) × (breath)

= 6.3 × 3.2 = 20.16 cm^2

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Division of a Decimal Number by another Decimal number

Example 1.40

Find $\frac{17.6}{0.4}$.

Solution

We have $17.6 \div 0.4 = \frac{176}{10} \div \frac{4}{10}$

$= \frac{176}{10} \times \frac{10}{4} = 44.$

Try these

Find :

i) $\frac{9.25}{0.5}$

ii) $\frac{36}{0.04}$

iii) $\frac{6.5}{1.3}$

Example 1.41

A car covers a distance of 129.92 km in 3.2 hours. What is the distance covered by it in 1 hour?

Solution

Distance covered by the car = 129.92 km.

Time required to cover this distance = 3.2 hours.

So, distance covered by it in 1 hour $= \frac{129.92}{3.2} = \frac{1299.2}{32} = 40.6$km.

60]

Example 1.42

Express 512 as a power .

Solution

We have $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

So we can say that $512 = 2^9$

Example: 1.43

Which one is greater 2^5 , 5^2 ?

Solution

We have $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

and $5^2 = 5 \times 5 = 25$

Since $32 > 25$.

Therefore 2^5 is greater than 5^2.

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1. Natural numbrs N = {1, 2, 3, ...}
2. Whole numbers W = {0, 1, 2, ...}
3. Integers Z = {..., – 3, – 2, – 1, 0, 1, 2, 3, ...}
4. The product of two positive integers is a positive integer.
5. The product of two negative integers is a positive integer.
6. The product of a positive integer and a negative integer is a negative integer.
7. The division of two integers need not be an integer.
8. Fraction is a part of whole.
9. If the product of two non-zero numbers is 1 then the numbers are called the reciprocal of each other.
10. $a \times a \times a \times \ldots$ m times $= a^m$
    (read as 'a' raised to the power m (or) the m^{th} power of 'a')
11. For any two non-zero integers a and b and whole numbers m and n,
    i) $a^m a^n = a^{m+n}$
    ii) $\frac{a^m}{a^n} = a^{m-n}$, where m > n
    iii) $(a^m)^n = a^{mn}$
    iv) $(-1)^n = 1$, when n is an even number
    $(-1)^n = -1$, when n is an odd number

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Example 2.21

Subtract $6a - 3b$ from $-8a + 9b$.

Solution

$$\begin{array}{r} -8a + 9b \\ +6a - 3b \\ (-) \quad (+) \\ \hline -14a + 12b \\ \hline \end{array}$$

Example 2.22

Subtract $2(p - q)$ from $3(5p - q + 3)$

Solution $3(5p - q + 3) - 2(p - q)$

$= 15p - 3q + 9 - 2p + 2q$

$= 15p - 2p - 3q + 2q + 9$

$= 13p - q + 9$

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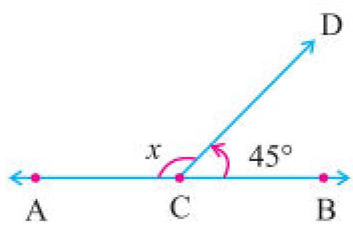
Example 3.4

Find the value of x in the given figure.



Solution

$\angle BCD + \angle DCA = 180^{\circ}$

(Since $\angle BCA = 180^{\circ}$ is a straight angle)

$45^{\circ} + x = 180^{\circ}$

$x = 180^{\circ} - 45^{\circ}$

$= 135^{\circ}$

$\therefore$ The value of x is 135°.

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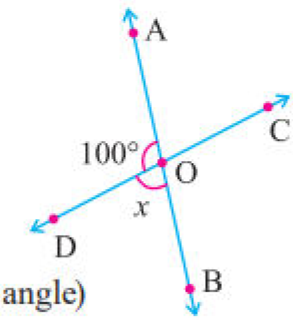
Example 3.5



Find the value of x in the given figure.

Solution

$$\angle AOD + \angle DOB = 180^{0}$$

(Since $\angle AOB = 180^{0}$ is a straight angle)

$$100^{0} + x = 180^{0}$$

$$x = 180^{0} - 100^{0}$$

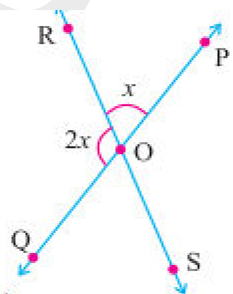
$$= 80^{0}$$

$\therefore$ The value of x is 80^{0}.

65]

$\therefore$ The value of x is 80^{0}.

Example 3.6



Find the value of x in the given figure.

Solution

$$\angle POR + \angle ROQ = 180^{0}$$

(Since $\angle POQ = 180^{0}$ is a straight angle)

$$x + 2x = 180^{0}$$

$$3x = 180^{0}$$

$$x = \frac{180^{0}}{3}$$

$$= 60^{0}$$

$\therefore$ The value of x is 60^{0}

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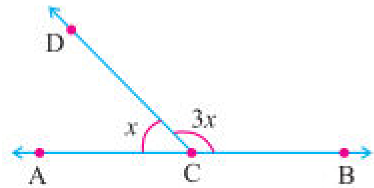
Example 3.7

Find the value of x in the given figure.



Solution

$\angle BCD + \angle DCA = 180^0$

(Since $\angle BCA = 180^0$ is a straight angle)

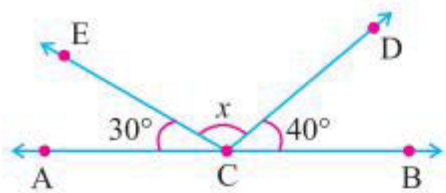
$$3x + x = 180^0$$
$$4x = 180^0$$
$$x = \frac{180^0}{4}$$
$$= 45^0$$

$\therefore$ The value of x is 45^0

67]

Example 3.8

Find the value of x in the given figure.



Solution

$\angle BCD + \angle DCE + \angle ECA = 180^0$

(Since $\angle BCA = 180^0$ is a straight angle)

$$40^0 + x + 30^0 = 180^0$$
$$x + 70^0 = 180^0$$
$$x = 180^0 - 70^0$$
$$= 110^0$$

$\therefore$ The value of x is 110^0

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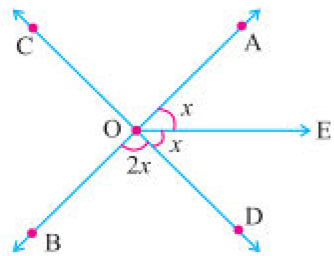
Example 3.11

Find the value of x in the given figure.



Solution

$\angle BOD + \angle DOE + \angle EOA = 180^\circ$

(Since $\angle$ AOB = 180° is straight angle)

$$2x + x + x = 180^\circ$$

$$4x = 180^\circ$$

$$x = \frac{180^\circ}{4}$$

$$= 45^\circ$$

$\therefore$ The value of x is 45°

69]

Example 1.1:

Find 5 equivalent ratios of 2:7

Solution: 2 : 7 can be written as $\frac{2}{7}$.

Multiplying the numerator and the denominator of $\frac{2}{7}$ by 2, 3, 4, 5, 6

we get $\frac{2\times2}{7\times2} = \frac{4}{14}, \frac{2\times3}{7\times3} = \frac{6}{21}, \frac{2\times4}{7\times4} = \frac{8}{28}$

$\frac{2\times5}{7\times5} = \frac{10}{35}, \quad \frac{2\times6}{7\times6} = \frac{12}{42}$

4 : 14, 6 : 21, 8 : 28, 10 : 35, 12 : 42 are equivalent ratios of 2 : 7.

Example 1.2:

Reduce 270 : 378 to its lowest term.

Solution:

$270:378 = \frac{270}{378}$

Dividing both the numerator and the denominator by 2, we get

$\frac{270\div2}{378\div2} = \frac{135}{189}$

by 3, we get

$\frac{135\div3}{189\div3} = \frac{45}{63}$

by 9, we get

$\frac{45\div9}{63\div9} = \frac{5}{7}$

270 : 378 is reduced to 5 : 7

Aliter:

Factorizing 270,378 we get

$$\frac{270}{378} = \frac{2\times3\times3\times3\times5}{2\times3\times3\times3\times7}$$

$$= \frac{5}{7}$$

70]

Example 1.3

Find the ratio of 9 months to 1 year

Solution: 1 year = 12 months

Ratio of 9 months to 12 months = 9 : 12

9 : 12 can be written as $\frac{9}{12}$

$= \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$

= 3 : 4

71]

Example 1.4

If a class has 60 students and the ratio of boys to girls is 2:1, find the number of boys and girls.

Solution:

Number of students = 60

Ratio of boys to girls = 2 : 1

Total parts = 2 + 1 = 3

Number of boys = $\frac{2}{3}$ of 60

$= \frac{2}{3} \times 60 = 40$

Number of boys = 40

Number of girls = Total Number of students – Number of boys

= 60 – 40

= 20

Number of girls = 20

[OR]

Number of girls

$= \frac{1}{3}$ of $60 = \frac{1}{3} \times 60$

= 20

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Example 1.5

A ribbon is cut into 3 pieces in the ratio 3: 2: 7. If the total length of the ribbon is 24 m, find the length of each piece.

Solution:

Length of the ribbon = 24m

Ratio of the 3 pieces = 3 : 2 : 7

Total parts = 3 + 2 + 7 = 12

Length of the first piece of ribbon = $\frac{3}{12}$ of 24

$= \frac{3}{12} \times 24 = 6$ m

Length of the second piece of ribbon = $\frac{2}{12}$ of 24

$= \frac{2}{12} \times 24 = 4$ m

Length of the last piece of ribbon = $\frac{7}{12}$ of 24

$= \frac{7}{12} \times 24 = 14$ m

So, the length of the three pieces of ribbon are 6 m, 4 m, 14 m respectively.

73]

Example 1.6

The ratio of boys to girls in a class is 4 : 5. If the number of boys is 20, find the number of girls.

Solution: Ratio of boys to girls = 4 : 5

Number of boys = 20

Let the number of girls be x

The ratio of the number of boys to the number of girls is $20 : x$

$4 : 5$ and $20 : x$ are in proportion, as both the ratios represent the number of boys and girls.

(i.e.) $4 : 5 :: 20 : x$

$$\text{Product of extremes} = 4 \times x$$

$$\text{Product of means} = 5 \times 20$$

In a proportion, product of extremes = product of means

$$4 \times x = 5 \times 20$$

$$x = \frac{5 \times 20}{4} = 25$$

Number of girls = 25

74]

Example 1.7

If A : B = 4 : 6, B : C = 18 : 5, find the ratio of A : B : C.

Solution:

A : B = 4 : 6

B : C = 18 : 5

L.C.M. of 6, 18 = 18

A : B = 12 : 18

B : C = 18 : 5

A : B : C =12 : 18 : 5

HINT

To compare 3 ratios as given in the example, the consequent (2nd term) of the 1st ratio and the antecedent (1st term) of the 2nd ratio must be made equal.

75]

Look at the table given below:

Number of pens	x	2	4	7	10	20
Cost of pens (₹)	y	100	200	350	500	1000

We see that as 'x' increases (↑) 'y' also increases (↑).

We shall find the ratio of number of pens to cost of pens

$$\frac{\text{Number of pens}}{\text{Cost of pens}} = \frac{x}{y}, \text{ to be } \frac{2}{100}, \frac{4}{200}, \frac{7}{350}, \frac{10}{500}, \frac{20}{1000}$$

and we see that each ratio = $\frac{1}{50}$ = Constant.

Ratio of number of pens to cost of pens is a constant.

$$\therefore \frac{x}{y} = \text{constant}$$

It can be said that **when two quantities vary directly the ratio of the two given quantities is always a constant**.

Now, look at the example given below:

Time taken (Hrs)	$x_1 = 2$	$x_2 = 10$
Distance travelled (km)	$y_1 = 10$	$y_2 = 50$

We see that as time taken increases (↑), distance travelled also increases (↑).

$$X = \frac{x_1}{x_2} = \frac{2}{10} = \frac{1}{5}$$

$$Y = \frac{y_1}{y_2} = \frac{10}{50} = \frac{1}{5}$$

$$X = Y = \frac{1}{5}$$

From the above example, it is clear that **in direct variation, when a given quantity is changed in some ratio then the other quantity is also changed in the same ratio**.

Now, study the relation between the given variables and find a and b.

Time taken (hrs)	x	2	5	6	8	10	12
Distance travelled (Km)	y	120	300	a	480	600	b

Here again, we find that the ratio of the time taken to the distance travelled is a constant.

$$\frac{\text{Time taken}}{\text{Distance travelled}} = \frac{2}{120} = \frac{5}{300} = \frac{10}{600} = \frac{8}{480} = \frac{1}{60} = \text{Constant}$$

(i.e.) $\frac{x}{y} = \frac{1}{60}$. Now, we try to find the unknown

$$\frac{1}{60} = \frac{6}{a}$$

$$\frac{1 \times \boxed{6}}{60 \times \boxed{6}} = \frac{6}{360}$$

$$a = 360$$

$$\frac{1}{60} = \frac{12}{b}$$

$$\frac{1 \times \boxed{12}}{60 \times \boxed{12}} = \frac{12}{720}$$

$$b = 720$$

76]

Example 1.8

If the cost of 16 pencils is ₹48, find the cost of 4 pencils.

Solution:

Let the cost of four pencils be represented as 'a'.

Number of pencils	Cost (₹)
x	y
16	48
4	a

As the number of pencils decreases (↓), the cost also decreases (↓). Hence the two quantities are in **direct variation**.

We know that, in direct variation, $\frac{x}{y}$ = constant

$$\frac{16}{48} = \frac{4}{a}$$

$$16 \times a = 48 \times 4$$

$$a = \frac{48 \times 4}{16} = 12$$

Cost of four pencils = ₹12

Aliter:

Let the cost of four pencils be represented as 'a' .

Number of pencils	Cost (₹)
x	y
16	48
4	a

As number of pencils decreases (↓), cost also decreases (↓), **direct variation (Same ratio)**.

$$\frac{16}{4} = \frac{48}{a}$$

$$16 \times a = 4 \times 48$$

$$a = \frac{4 \times 48}{16} = 12$$

Cost of four pencils = ₹12.

Example 1.9

A car travels 360 km in 4 hrs. Find the distance it covers in 6 hours 30 mins at the same speed.

77]

Example 1.9

A car travels 360 km in 4 hrs. Find the distance it covers in 6 hours 30 mins at the same speed.

Solution:

Let the distance travelled in $6\frac{1}{2}$ hrs be a

Time taken (hrs)	**Distance travelled (km)**
x	y
4	360
$6\frac{1}{2}$	a

$$30 \text{ mins} = \frac{30}{60}\text{hrs} = \frac{1}{2}\text{of an hr}$$

$$6 \text{ hrs } 30 \text{ mins} = 6\frac{1}{2} \text{ hrs}$$

As time taken increases (↑), distance travelled also increases (↑), **direct variation.**

In direct variation, $\frac{x}{y}$ = constant

$$\frac{4}{360} = \frac{6½}{a}$$

$$4 \times a = 360 \times 6\frac{1}{2}$$

$$4 \times a = 360 \times \frac{13}{2}$$

$$a = \frac{360 \times 13}{4 \times 2} = 585$$

Distance travelled in $6\frac{1}{2}$ hrs = 585 km

78]

Aliter: Let the distance travelled in $6\frac{1}{2}$ hrs be a

Time taken (hrs)	Distance travelled (km)
4	360
$6\frac{1}{2}$	a

As time taken increases (↑), distance travelled also increases (↑), **direct variation (same ratio).**

$$\frac{4}{6\frac{1}{2}} = \frac{360}{a}$$
$$4 \times a = 360 \times 6\frac{1}{2}$$
$$4 \times a = 360 \times \frac{13}{2}$$
$$a = \frac{360}{4} \times \frac{13}{2} = 585$$

Distance travelled in $6\frac{1}{2}$ hrs = 585 km.

79]

7 men can complete a work in 52 days. In how many days will 13 men finish the same work?

Solution: Let the number of unknown days be a.

Number of men	**Number of days**
x	y
7	52
13	a

As the number of men increases (↑), number of days decreases (↓), **inverse variation**

In inverse variation, xy = constant

$$7 \times 52 = 13 \times a$$

$$13 \times a = 7 \times 52$$

$$a = \frac{7 \times 52}{13} = 28$$

13 men can complete the work in 28 days.

Aliter:

Let the number of unknown days be a.

Number of men	**Number of days**
7	52
13	a

As number of men increases (↑), number of days decreases (↓), **inverse variation (inverse ratio).**

$$\frac{7}{13} = \frac{a}{52}$$

$$7 \times 52 = 13 \times a$$

$$13 \times a = 7 \times 52$$

$$a = \frac{7 \times 52}{13} = 28$$

13 men can complete the work in 28 days

80]

Example 1.11

A book contains 120 pages. Each page has 35 lines . How many pages will the book contain if every page has 24 lines per page?

Solution: Let the number of pages be a.

Number of lines per page	Number of pages
35	120
24	a

As the number of lines per page decreases (↓) number of pages increases (↑) it is in **inverse variation (inverse ratio)**.

$$\frac{35}{24} = \frac{a}{120}$$

$$35 \times 120 = a \times 24$$

$$a \times 24 = 35 \times 120$$

$$a = \frac{35 \times 120}{24}$$

$$a = 35 \times 5 = 175$$

If there are 24 lines in one page, then the number of pages in the book = 175

81]

Example 2.1

Find the area and the perimeter of a rectangular field of length 15 m and breadth 10 m.

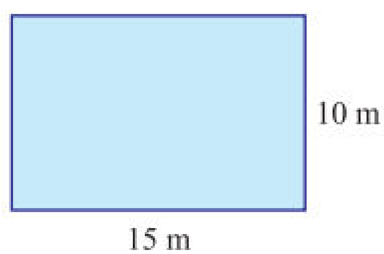


Fig.2.3

Solution

Given: length = 15 m and breadth = 10 m

Area of the rectangle = length × breadth

= 15 m × 10 m

= 150 m^2

$$\text{Perimeter of the rectangle} = 2\,[\text{length} + \text{breadth}]$$
$$= 2\,[15 + 10] = 50 \text{ m}$$
$$\therefore \text{ Area of the rectangle} = 150 \text{ m}^2$$
$$\text{Perimeter of the rectangle} = 50 \text{ m}$$

82]

Example 2.2

The area of a rectangular garden 80m long is 3200sq.m. Find the width of the garden.

Solution

Given: length = 80 m, Area = 3200 sq.m

$$\text{Area of the rectangle} = \text{length} \times \text{breadth}$$
$$\text{breadth} = \frac{\text{area}}{\text{length}}$$
$$= \frac{3200}{80} = 40 \text{ m}$$
$$\therefore \text{ Width of the garden} = 40 \text{ m}$$

83]

Example 2.6

In a right triangular ground, the sides adjacent to the right angle are 50 m and 80 m. Find the cost of cementing the ground at ₹5 per sq.m

Solution

For finding the cost of cementing, we need to find the area and then multiply it by the rate for cementing.

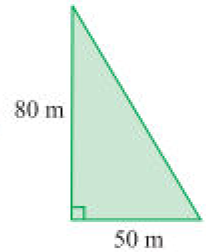


Fig. 2.5

Area of right triangular ground = $\frac{1}{2} \times b \times h$

where b and h are adjacent sides of the right anlges.

$$= \frac{1}{2} \times (50\, m \times 80\, m)$$
$$= 2000 \text{ m}^2$$

cost of cementing one sq.m = ₹5

∴ cost of cementing 2000 sq.m = ₹5 × 2000

= ₹10000

Do you know?

1 are = 100 m^2

1 hectare = 100 are (or)

= 10000 m^2

84]

Example 2.7

Find the area of the adjacent figure:

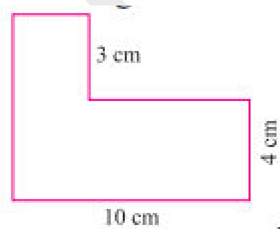


Fig. 2.8

Solution

Area of square (1) = 3 cm × 3 cm = 9 cm^2

Area of rectangle (2) = 10 cm × 4 cm = 40 cm^2

∴ Total area of the figure (Fig. 2.9) = (9 + 40) cm^2

= 49 cm^2

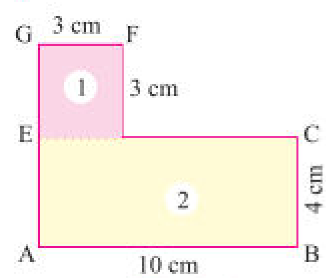


Fig. 2.9

Aliter:

Area of rectangle (1) = 7 cm × 3 cm = 21 cm^2

Area of rectangle (2) = 7 cm × 4 cm = 28 cm^2

∴ Total area of the figure (Fig. 2.10) = (21 + 28) cm^2

= 49 cm^2

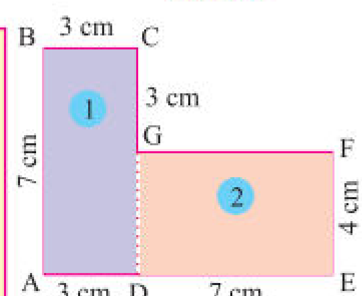


Fig. 2.10

85]

Example 2.8

Find the area of the following figure:

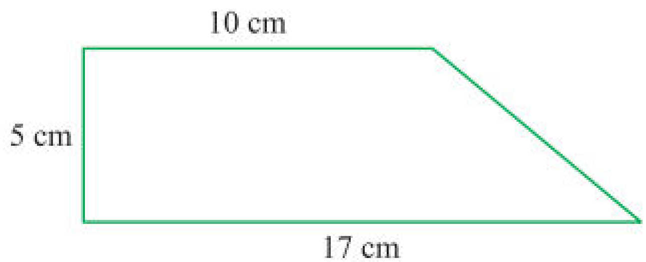


Fig. 2.11

Solution

The figure contains a rectangle and a right triangle

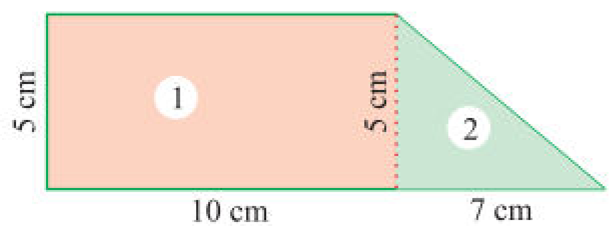


Fig. 2.12

$$\text{Area of the rectangle (1)} = 5\text{ cm} \times 10\text{ cm}$$
$$= 50\text{ cm}^2$$
$$\text{Area of the right triangle (2)} = \frac{1}{2} \times (7\text{ cm} \times 5\text{ cm})$$
$$= \frac{35}{2}\text{ cm}^2 = 17.5\text{ cm}^2$$
$$\therefore \text{Total area of the figure} = (50 + 17.5)\text{ cm}^2$$
$$= 67.5\text{ cm}^2$$
$$\text{Total area} = 67.5\text{ cm}^2$$

86]

2.3 Area of Triangle

The area of a right triangle is half the area of the rectangle that contains it.

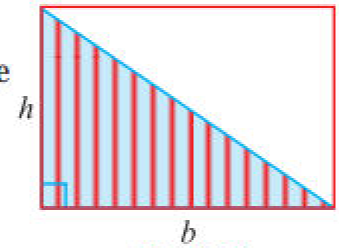


Fig. 2.14

The area of the right triangle

$= \frac{1}{2}$(Product of the sides containing 90^{0})

(or) $= \frac{1}{2} b\, h$ sq.units

where b and h are adjacent sides of the right triangle.

In this section we will learn to find the area of triangles.

87]

Example 2.10

Find the area of the following figures:

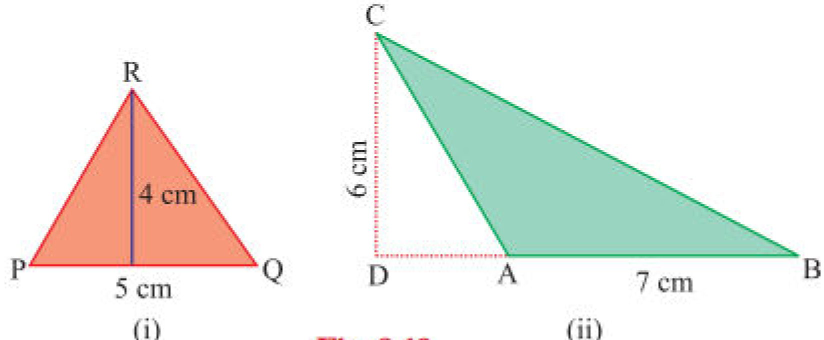


Fig. 2.19

Solution

(i) Given: Base $=$ 5 cm, Height $=$ 4 cm

Area of the triangle PQR $= \frac{1}{2} b\, h$

$= \frac{1}{2} \times 5 \text{ cm} \times 4 \text{ cm}$

$=$ 10 sq.cm (or) cm^2

(ii) Given: Base $=$ 7cm, Height $=$ 6cm

Area of the triangle ABC $= \frac{1}{2} b\, h$

$= \frac{1}{2} \times 7\text{cm} \times 6\text{cm}$

$=$ 21 sq.cm (or) cm^2

88]

Example 2.11

Area of a triangular garden is 800 sq.m. The height of the garden is 40 m. Find the base length of the garden.

Solution

$$\text{Area of the triangular garden} = 800 \text{ sq.m. (given)}$$

$$\frac{1}{2} b\, h = 800$$

$$\frac{1}{2} \times b \times 40 = 800 \quad (\text{since } h = 40)$$

$$20\, b = 800$$

$$b = 40 \text{ m}$$

$\therefore$ Base of the garden is 40 m.

89]

Example 2.12

Calculate the area of a quadrilateral PQRS shown in the figure

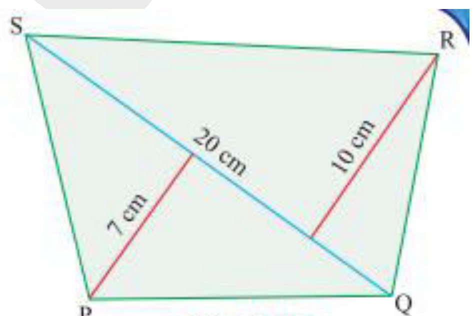


Fig. 2.23

Solution

Given: $d = 20\text{cm}$, $h_1 = 7\text{cm}$, $h_2 = 10\text{cm}$.

Area of a quadrilateral PQRS

$$= \frac{1}{2} \times d \times (h_1 + h_2)$$

$$= \frac{1}{2} \times 20 \times (7 + 10)$$

$$= 10 \times 17$$

$$= 170 \text{ cm}^2$$

$\therefore$ Area of the quadrilateral PQRS = 170 cm^2.

90]

Example 2.16

Using the data given in the figure,

(i) find the area of the parallelogram with base AB.

(ii) find the area of the parallelogram with base AD.

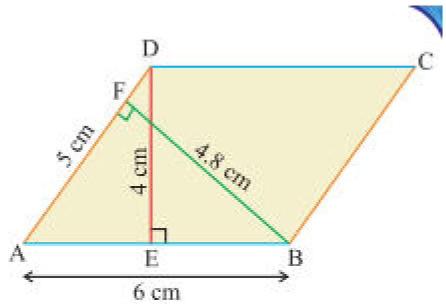


Fig. 2.30

Solution

The area of the parallelogram = base × height

(i) Area of parallelogram with base AB = base AB × height DE

= 6 cm × 4 cm

= 24 cm^2

(ii) Area of parallelogram with base AD = base AD × height FB

= 5 cm × 4.8 cm

= 24 cm^2

91]

Area of the rhombus in terms of its diagonals

In a rhombus ABCD , AB ∥ DC and BC ∥ AD

Also, AB = BC = CD = DA

Let the diagonals be d_1 (AC) and d_2 (BD)

Since, the diagonals bisect each other at right angles

AC ⊥ BD and BD ⊥ AC

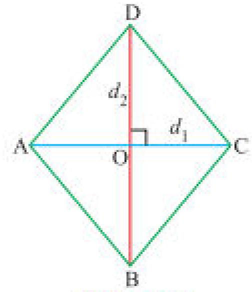


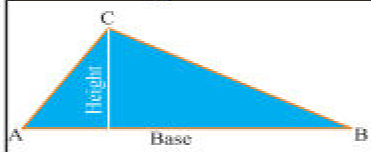
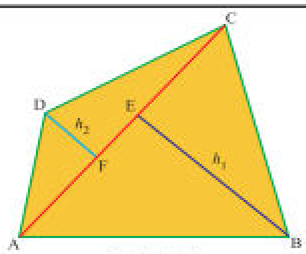
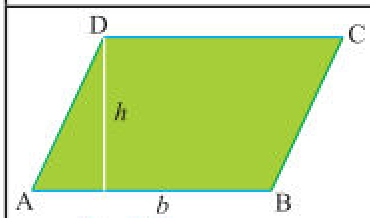
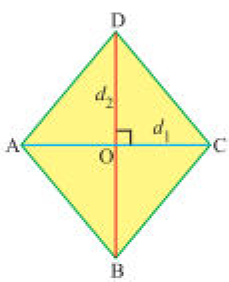
Fig. 2.34

Area of the rhombus ABCD

$$= \text{Area of } \Delta\, ABC + \text{Area of } \Delta ADC$$
$$= \left[\frac{1}{2} \times AC \times OB\right] + \left[\frac{1}{2} \times AC \times OD\right]$$
$$= \frac{1}{2} \times AC \times (OB + OD)$$
$$= \frac{1}{2} \times AC \times BD$$
$$= \frac{1}{2} \times d_1 \times d_2 \text{ sq. units}$$

$$\therefore \text{Area of the rhombus} = \frac{1}{2}[d_1 \times d_2] \text{ sq. units}$$
$$= \frac{1}{2} \times (\text{product of diagonals}) \text{ sq. units}$$

92]

Figure	Area	Formula
 Triangle	$\frac{1}{2} \times$ base $\times$ height	$\frac{1}{2} \times b \times h$ sq. units.
 Quadrilateral	$\frac{1}{2} \times$ diagonal $\times$ (sum of the perpendicular distances drawn to the diagonal from the opposite vertices)	$\frac{1}{2} \times d \times (h_1 + h_2)$ sq. units
 Parallelogram	base $\times$ corresponding altitude	bh sq. units
 Rhombus	$\frac{1}{2} \times$ product of diagonals	$\frac{1}{2} \times d_1 \times d_2$ sq. units

93]

Example 1.10

Sum of three consecutive integers is 45. Find the integers.

Solution: Let the first integer be x.

$\Rightarrow$ second integer $= x + 1$

Third integer $= x + 1 + 1 = x + 2$

Their sum $= x + (x + 1) + (x + 2) = 45$

$$3x + 3 = 45$$
$$3x = 42$$
$$x = 14$$

Hence, the integers are $x = 14$

$$x + 1 = 15$$
$$x + 2 = 16$$

94]

Example 1.11

A number when added to 60 gives 75. What is the number?

Solution: Let the number be x.

The equation is $60 + x = 75$

$$x = 75 - 60$$
$$x = 15$$

Example 1.12

20 less than a number is 80. What is the number?

Solution: Let the number be x.

The equation is $x - 20 = 80$

$$x = 80 + 20$$
$$x = 100$$

95]

Example 1.13

$\frac{1}{10}$ of a number is 63. What is the number?

Solution: Let the number be x.

The equation is $\frac{1}{10}(x) = 63$

$$\frac{1}{10}(x) \times 10 = 63 \times 10$$

$$x = 630$$

Example 1.14

A number divided by 4 and increased by 6 gives 10. Find the number.

Solution: Let the number be x.

The equation is $\frac{x}{4} + 6 = 10$

$$\frac{x}{4} = 10 - 6$$

$$\frac{x}{4} = 4$$

$$\frac{x}{4} \times 4 = 4 \times 4$$

$\therefore$ the number is 16.

96]

Example 1.15

Thendral's age is 3 less than that of Revathi. If Thendral's age is 18, what is Revathi' s age?

Solution: Let Revathi's age be x

$\Rightarrow$ Thendral's age $= x - 3$

Given, Thendral's age is 18 years

$$\Rightarrow \quad x - 3 = 18$$

$$x = 18 + 3$$

$$x = 21$$

Hence Revathi's age is 21 years.

97]

Example 2.1

Express $\frac{3}{5}$ as a percent

Solution:

5 multiplied by 20 gives 100

$$\frac{3 \times 20}{5 \times 20} = \frac{60}{100} = 60\%$$

$$\frac{3}{5} = 60\%$$

Example 2.2

Express $6\frac{1}{4}$ as a percent

Solution:

$$6\frac{1}{4} = \frac{25}{4}$$

4 multiplied by 25 gives 100

$$\frac{25 \times 25}{4 \times 25} = \frac{625}{100} = 625\%$$

98]

(ii) Fractions with denominators that cannot be converted to 100

Example 2.3

Express $\frac{4}{7}$ as a percent

Solution: Multiply by 100%

$$\left(\frac{4}{7} \times 100\right)\% = \frac{400}{7}\%$$

$$= 57\frac{1}{7}\% = 57.14\%$$

Example 2.4

Express $\frac{1}{3}$ as a percent

Solution: Multiply by 100%

$$\left(\frac{1}{3} \times 100\right)\% = \left(\frac{100}{3}\right)\%$$

$$= 33\tfrac{1}{3}\% \text{ (or) } 33.33\%$$

99]

Example 2.5

There are 250 students in a school. 55 students like basketball, 75 students like football, 63 students like throw ball, while the remaining like cricket. What is the percent of students who like (a) basket ball? (b) throw ball?

Solution:

Total number of students = 250

(a) Number of students who like basket ball = 55

55 out of 250 like basket ball which can be represented as $\frac{55}{250}$

Percentage of students who like basket ball = $\left(\frac{55}{250}\times 100\right)\%$

$= 22\%$

(b) Number of students who like throw ball = 63

63 out of 250 like throw ball and that can be represented as $\frac{63}{250}$

Percentage of students who like throw ball = $\left(\frac{63}{250}\times 100\right)\%$

$= \frac{126}{5}\% = 25.2\%$

22% like basket ball, 25.2% like throw ball.

100]

(iii) To convert decimals to percents

Example 2.6

Express 0.07 as a percent

Solution:

Multiply by 100%

$(0.07 \times 100)\% = 7\%$

Aliter:

$$0.07 = \frac{7}{100} = 7\%$$

Example 2.7

Express 0.567 as a percent

Solution:

Multiply by 100%

$(0.567 \times 100)\% = 56.7\%$

Aliter: $0.567 = \frac{567}{1000} = \frac{567}{10 \times 100}$

$$= \frac{56.7}{100} = 56.7\%$$

Note: To convert a fraction or a decimal to a percent, multiply by 100%.

101]

Example 2.8

Express 12% as a fraction.

Solution:

$$12\% = \frac{12}{100} \text{(reduce the fraction to its lowest terms)}$$

$$= \frac{3}{25}$$

102]

Example 2.9

Express $233\frac{1}{3}\%$ as a fraction.

Solution:

$$233\tfrac{1}{3}\% = \frac{700}{3}\%$$
$$= \frac{700}{3 \times 100} = \frac{7}{3}$$
$$= 2\tfrac{1}{3}$$

Percents that have easy fractions

$$50\% = \frac{1}{2}$$
$$25\% = \frac{1}{4}$$
$$33\frac{1}{3}\% = \frac{1}{3}$$

Find more of this kind

Example 2.10

Express $\frac{1}{4}\%$ as a fraction

Solution:

$$\frac{1}{4}\% = \frac{1}{4 \times 100} = \frac{1}{400}$$

(ii) A percent is a fraction with its denominator 100. To convert this fraction to a decimal, take the numerator and move the decimal point to its left by 2 digits.

Example 2.11

Express 15% as a decimal.

Solution:

$$15\% = \frac{15}{100} = 0.15$$

103]

Example 2.12

Express 25.7% as a decimal.

Solution:

$$25.7\% = \frac{25.7}{100}$$
$$= 0.257$$

104]

Example 2.14

Find the value of $\frac{1}{2}\%$ of 200.

Solution:

$$= \frac{\frac{1}{2}}{100} \text{ of } 200$$

$$= \frac{1}{2 \times 100} \times 200$$

$$\frac{1}{200} \times 200 = 1$$

$$\frac{1}{2}\% \text{ of } 200 = 1$$

105]

Example 2.15

Find the value of 0.75% of 40 kg.

Solution:

$$0.75\% = \frac{0.75}{100}$$

$$0.75\% \text{ of } 40 = \frac{0.75}{100} \times 40$$

$$= \frac{3}{10} = 0.3$$

$$0.75\% \text{ of } 40\text{kg} = 0.3\text{kg.}$$

106]

Example 2.16

In a class of 70, 60% are boys. Find the number of boys and girls.

Solution:

Total number of students $= 70$

Number of boys $= 60\%$ of 70

$= \frac{60}{100} \times 70$

$= 42$

Number of boys $= 42$

Number of girls $=$ Total students – Number of boys

$= 70 - 42$

$= 28$

Number of girls $= 28$

107]

Example 2.17

In 2010, the population of a town is 1,50,000. If it is increased by 10% in the next year, find the population in 2011.

Solution:

Population in 2010 $= 1,50,000$

Increase in population $= \frac{10}{100} \times 1,50,000$

$= 15,000$

Population in 2011 $= 150000 + 15000$

$= 1,65,000$

108]

- When the selling price of an article is greater than its cost price, then there is a profit.

  Profit = Selling Price – Cost Price
- When the cost price of an article is greater than its selling price, then there is a loss.

  Loss = Cost Price – Selling Price
- S.P = C.P + Profit
- S.P = C.P – Loss.

109]

$$\boxed{\text{So, Profit Percentage} = \frac{\text{Profit}}{\text{C.P}} \times 100}$$

Loss % is also calculated in the same way.

$$\boxed{\text{Loss Percentage} = \frac{\text{Loss}}{\text{C.P.}} \times 100}$$

Profit Percentage or Loss Percentage is always calculated on the cost price of the article.

110]

Example 2.18

A dealer bought a television set for ₹10,000 and sold it for ₹12,000. Find the profit / loss made by him for 1 television set. If he had sold 5 television sets, find the total profit/loss

Solution:

Selling Price of the television set = ₹12,000

Cost Price of the television set = ₹10,000

S.P. > C.P, there is a profit

Profit = S.P. – C. P.

= 12000 – 10000

Profit = ₹2,000

Profit on 1 television set = ₹2,000

Profit on 5 television sets = 2000×5

Profit on 5 television sets = ₹10,000

111]

Example 2.19

Sanjay bought a bicycle for ₹5,000. He sold it for ₹600 less after two years. Find the selling price and the loss percent.

Solution:

Cost Price of the bicycle = ₹5000

$$\begin{aligned}
\text{Loss} &= ₹600 \\
\text{Selling Price} &= \text{Cost Price} - \text{Loss} \\
&= 5000 - 600 \\
\text{Selling Price of the bicycle} &= ₹4400 \\
\text{Loss} &= \frac{\text{Loss}}{\text{C.P.}} \times 100 \\
&= \frac{600}{5000} \times 100 \\
&= 12\% \\
\text{Loss} &= 12\%
\end{aligned}$$

112]

Example 2.20

A man bought an old bicycle for ₹1,250. He spent ₹250 on its repairs. He then sold it for ₹1400. Find his gain or loss %

Solution:

Cost Price of the bicycle = ₹1,250

Repair Charges = ₹250

Total Cost Price = 1250 + 250 = ₹1,500

Selling Price = ₹1,400

C.P. > S.P., there is a Loss

Loss = Cost Price – Selling Price

= 1500 – 1400

= 100

Loss = ₹100

Percentage of the loss $= \frac{\text{Loss}}{\text{C.P.}} \times 100$

$= \frac{100}{1500} \times 100$

$= \frac{20}{3}$

$= 6\frac{2}{3}\%$ (or) 6.67%

Loss = 6.67%

113]

Example 2.21

A fruit seller bought 8 boxes of grapes at ₹150 each. One box was damaged. He sold the remaining boxes at ₹190 each. Find the profit / loss percent.

Solution:

Cost Price of 1 box of grapes = ₹150

Cost Price of 8 boxes of grapes = 150×8

= ₹1200

Number of boxes damaged = 1

Number of boxes sold = $8 - 1$

= 7

Selling Price of 1 box of grapes = ₹190

Selling Price of 7 boxes of grapes = 190×7

= ₹1330

S.P. > C.P, there is a Profit.

Profit = Selling Price – Cost Price

= $1330 - 1200$

= 130

Profit = ₹130

$$\text{Percentage of the profit} = \frac{\text{Profit}}{\text{C.P}} \times 100$$

$$= \frac{130}{1200} \times 100$$

$$= 10.83$$

Profit = 10.83 %

114]

Example 2.22

Ram, the shopkeeper bought a pen for ₹50 and then sold it at a loss of ₹5. Find his selling price.

Solution:

Cost price of the pen = ₹50

Loss = ₹5

S.P. = C.P. – Loss

= 50 – 5

= 45

Selling price of the pen = ₹45.

115]

Example 2.23

Sara baked cakes for the school festival. The cost of one cake was ₹55. She sold 25 cakes and made a profit of ₹11 on each cake. Find the selling price of the cakes and the profit percent.

Solution:

Cost price of 1 cake = ₹55

Number of cakes sold = 25

Cost price of 25 cakes = $55 \times 25 =$ ₹1375

Profit on 1 cake = ₹11

Profit on 25 cakes = $11 \times 25 =$ ₹275

S.P. = C.P. + Profit

= $1375 + 275$

= 1,650

= ₹1,650

Percentage of the profit $= \frac{\text{Profit}}{\text{C. P}} \times 100$

$= \frac{275}{1375} \times 100$

= 20

Profit = 20 %

116]

Calculation of Interest

If 'r' is the rate of interest, principal is ₹100, then Interest

for 1 year $= 100 \times 1 \times \frac{r}{100}$

for 2 years $= 100 \times 2 \times \frac{r}{100}$

for 3 years $= 100 \times 3 \times \frac{r}{100}$

for n years $= 100 \times n \times \frac{r}{100}$

$$\text{So,} \quad I = \frac{Pnr}{100}$$

$$A = P + I$$

$$A = P + \frac{Pnr}{100}$$

$$A = P\left(1 + \frac{nr}{100}\right)$$

$$\text{Interest} = \text{Amount} - \text{Principal}$$

$$I = A - P$$

The other formulae derived from

$$I = \frac{Pnr}{100} \text{ are}$$

$$r = \frac{100I}{Pn}$$

$$n = \frac{100I}{Pr}$$

$$P = \frac{100I}{rn}$$

Note: 'n' is always calculated in years. When 'n' is given in months \ days, convert it into years.

117]

Example 2.24

Kamal invested ₹3,000 for 1 year at 7 % per annum. Find the simple interest and the amount received by him at the end of one year.

Solution:

Principal (P) = ₹3,000

Number of years (n) = 1

Rate of interest (r) = 7 %

$$\text{Interest (I)} = \frac{Pnr}{100}$$

$$= \frac{3000 \times 1 \times 7}{100}$$

$$I = ₹210$$

$$A = P + I$$

$$= 3000 + 210$$

$$A = ₹3,210$$

118]

Example 2.25

Radhika invested ₹5,000 for 2 years at 11 % per annum. Find the simple interest and the amount received by him at the end of 2 years.

Solution:

$$\text{Principal (P)} = ₹5,000$$

$$\text{Number of years } (n) = 2 \text{ years}$$

$$\text{Rate of interest } (r) = 11\ \%$$

$$I = \frac{Pnr}{100}$$

$$= \frac{5000 \times 2 \times 11}{100}$$

$$= 1100$$

$$I = ₹1,100$$

$$\text{Amount (A)} = P + I$$

$$= 5000 + 1100$$

$$A = ₹6,100$$

119]

Find the simple interest and the amount due on ₹7,500 at 8 % per annum for 1 year 6 months.

Solution:

Know this

12 months = 1 years

6 months = $\frac{6}{12}$ year

$= \frac{1}{2}$ year

3 months = $\frac{3}{12}$ year

$= \frac{1}{4}$ year

$$P = ₹7{,}500$$

$$n = 1 \text{ yr } 6 \text{ months}$$

$$= 1\frac{6}{12} yrs$$

$$= 1\frac{1}{2} = \frac{3}{2} yrs$$

$$r = 8\ \%$$

$$I = \frac{Pnr}{100}$$

$$= \frac{7500 \times \frac{3}{2} \times 8}{100}$$

$$= \frac{7500 \times 3 \times 8}{2 \times 100}$$

$$= 900$$

$$I = ₹900$$

$$A = P + I$$

$$= 7500 + 900$$

$$= ₹8{,}400$$

$$\text{Interest} = ₹900, \ \text{Amount} = ₹8{,}400$$

Aliter:

$$
\begin{aligned}
P &= ₹7,500 \\
n &= \frac{3}{2} \text{ years} \\
r &= 8\ \% \\
A &= P\left(1+\frac{nr}{100}\right) \\
&= 7500\left(1+\frac{\frac{3}{2}\times 8}{100}\right) \\
&= 7500\left(1+\frac{3\times 8}{2\times 100}\right) \\
&= 7500\left(\frac{28}{25}\right) \\
&= 300\times 28 \\
&= 8400 \\
A &= ₹8400 \\
I &= A - P \\
&= 8400 - 7500 \\
&= 900 \\
I &= ₹900 \\
\text{Interest} &= ₹900 \\
\text{Amount} &= ₹8,400
\end{aligned}
$$

120]

Example 2.27

Find the simple interest and the amount due on ₹6,750 for 219 days at 10 % per annum.

Solution:

$$P = ₹6{,}750$$

$$n = 219 \text{ days}$$

$$= \frac{219}{365} \text{ year} = \frac{3}{5} \text{ year}$$

$$r = 10\ \%$$

$$I = \frac{Pnr}{100}$$

$$I = \frac{6750 \times 3 \times 10}{5 \times 100}$$

$$= 405$$

$$I = ₹405$$

$$A = P + I$$

$$= 6750 + 405$$

$$= 7{,}155$$

$$A = ₹7{,}155$$

$$\text{Interest} = ₹405, \text{ Amount} = ₹7{,}155$$

Know this

$$365 \text{ days} = 1 \text{ year}$$

$$219 \text{ days} = \frac{219}{365} \text{ year} = \frac{3}{5} \text{year}$$

$$73 \text{ days} = \frac{73}{365} \text{ year} = \frac{1}{5} \text{ year}$$

121]

Example 2.29

Find the rate percent per annum when a principal of ₹7,000 earns a S.I. of ₹1,680 in 16 months.

Solution:

$$P = ₹7{,}000$$
$$n = 16 \text{ months}$$
$$= \frac{16}{12} yr = \frac{4}{3} yr$$
$$I = ₹1{,}680$$
$$r = ?$$
$$r = \frac{100I}{Pn}$$
$$= \frac{100 \times 1680}{7000 \times \frac{4}{3}}$$
$$= \frac{100 \times 1680 \times 3}{7000 \times 4}$$
$$= 18$$
$$r = 18\ \%$$

122]

Example 2.30

Vijay invested ₹10,000 at the rate of 5 % simple interest per annum. He received ₹11,000 after some years. Find the number of years.

Solution:

$$A = ₹11{,}000$$
$$P = ₹10{,}000$$

$$r = 5\ \%$$

$$n = ?$$

$$\begin{aligned} I &= A - P \\ &= 11{,}000 - 10{,}000 \\ &= 1{,}000 \end{aligned}$$

$$I = ₹1000$$

$$\begin{aligned} n &= \frac{100\,I}{P\,r} \\ &= \frac{100 \times 1000}{10000 \times 5} \end{aligned}$$

$$n = 2 \text{ years.}$$

123]

Aliter:

$$\begin{aligned} A &= P\left(1 + \frac{nr}{100}\right) \\ 11000 &= 10000\left(1 + \frac{n \times 5}{100}\right) \\ \frac{11000}{10000} &= 1 + \frac{n}{20} \\ \frac{11}{10} &= \frac{20 + n}{20} \\ \frac{11}{10} \times 20 &= 20 + n \\ 22 &= 20 + n \\ 22 - 20 &= n \\ n &= 2 \text{ years} \end{aligned}$$

124]

Example 2.31

A sum of money triples itself at 8 % per annum over a certain time. Find the number of years.

Solution:

Let Principal be ₹P.

$$\text{Amount} = \text{triple the principal}$$
$$= ₹3\ P$$
$$r = 8\ \%$$
$$n = ?$$

$$I = A - P$$
$$= 3P - P$$
$$= 2P$$
$$I = ₹2\ P$$
$$n = \frac{100I}{Pr}$$
$$= \frac{100 \times 2P}{P \times 8}$$
$$n = 25 \text{ years}$$

Number of years = 25

Aliter:

Let Principal be ₹100

$$\text{Amount} = 3 \times 100$$
$$= ₹300$$
$$I = A - P$$
$$= 300 - 100$$
$$I = ₹200.$$
$$n = \frac{100I}{Pr} = \frac{100 \times 200}{100 \times 8}$$
$$n = \frac{200}{8} = 25$$

Number of years = 25.

125]

Example 2.32

A certain sum of money amounts to ₹10,080 in 5 years at 8 %. Find the principal.

Solution:

$$A = ₹10,080$$
$$n = 5 \text{ years}$$
$$r = 8\ \%$$
$$P = ?$$
$$A = P\left(1 + \frac{nr}{100}\right)$$
$$10080 = P\left(1 + \frac{5 \times 8}{100}\right)$$

$$10080 = P\left(\frac{7}{5}\right)$$
$$10080 \times \frac{5}{7} = P$$
$$7{,}200 = P$$
$$\text{Principal} = ₹7{,}200$$

126]

Example 2.33

A certain sum of money amounts to ₹8,880 in 6 years and ₹7,920 in 4 years respectively. Find the principal and rate percent.

Solution:

$$\begin{aligned}
\text{Amount at the end of 6 years} &= \text{Principal + interest for 6 years}\\
&= P + I_6 = 8880\\
\text{Amount at the end of 4 years} &= \text{Principal + Interest for 4 years}\\
&= P + I_4 = 7920\\
I_2 &= 8880 - 7920\\
&= 960\\
\text{Interest at the end of 2 years} &= ₹960\\
\text{Interest at the end of 1st year} &= \frac{960}{2}\\
&= 480\\
\text{Interest at the end of 4 years} &= 480 \times 4\\
&= 1{,}920\\
P + I_4 &= 7920\\
P + 1920 &= 7920\\
P &= 7920 - 1920\\
P &= 6{,}000\\
\text{Principal} &= ₹6{,}000\\
r &= \frac{100I}{pn}\\
&= \frac{100 \times 1920}{6000 \times 4}\\
r &= 8\ \%
\end{aligned}$$

127]

1. A fraction whose denominator is 100 or a ratio whose second term is 100 is termed as a percent.
2. Percent means per hundred, denoted by %
3. To convert a fraction or a decimal to a percent, multiply by 100.
4. The price at which an article is bought is called the cost price of an article.
5. The price at which an article is sold is called the selling price of an article.
6. If the selling price of an article is more than the cost price, there is a profit.
7. If the cost price of an article is more than the selling price, there is a loss.
8. Total cost price = Cost Price + Repair Charges / Transportation charges.
9. Profit or loss is always calculated for the same number of articles or same units.
10. Profit = Selling Price – Cost Price
11. Loss = Cost Price – Selling Price
12. Profit% = $\frac{\text{Profit}}{\text{C.P.}} \times 100$
13. Loss% = $\frac{\text{Loss}}{\text{C.P.}} \times 100$
14. Selling Price = Cost Price + Profit
15. Selling Price = Cost Price - Loss
16. Simple interest is $I = \frac{Pnr}{100}$

17. $A = P + I$

$= P + \frac{Pnr}{100}$

$= P\left(1 + \frac{nr}{100}\right)$

18. $I = A - P$

19. $P = \frac{100I}{nr}$

20. $r = \frac{100I}{Pn}$

21. $n = \frac{100I}{Pr}$

128]

Area of a trapezium

ABCD is a trapezium with parallel sides AB and DC measuring 'a' and 'b'. Let the distance between the two parallel sides be 'h'. The diagonal BD divides the trapezium into two triangles ABD and BCD.

Area of the trapezium

$= \text{area of } \Delta ABD + \text{area of } \Delta BCD$

$= \frac{1}{2} \times AB \times h + \frac{1}{2} \times DC \times h$

$= \frac{1}{2} \times h[AB + DC]$

$= \frac{1}{2} \times h[a + b]$ sq. units

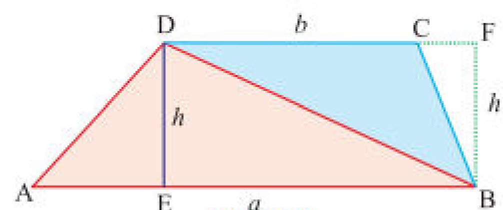


Fig. 3.3

$\therefore$ Area of a trapezium = $\frac{1}{2} \times$ height $\times$ (sum of the parallel sides) sq. units

129]

Example 3.1

Find the area of the trapezium whose height is 10 cm and the parallel sides are 12 cm and 8 cm of length.

Solution

Given: $h = 10$ cm, $a = 12$ cm, $b = 8$ cm

$$\text{Area of a trapezium} = \frac{1}{2} \times h(a + b)$$
$$= \frac{1}{2} \times 10 \times (12 + 8) = 5 \times (20)$$
$$\therefore \text{Area of the trapezium} = 100 \text{ sq. cm}^2$$

130]

Example 3.4

Find out the circumference of a circle whose diameter is 21 cm.

Solution

$$\text{Circumference of a circle} = \pi d$$
$$= \frac{22}{7} \times 21 \qquad \text{Here } \pi = \frac{22}{7}$$
$$= 66 \text{ cm.}$$

Example 3.5

Find out the circumference of a circle whose radius is 3.5 m.

Solution

$$\text{Circumference of a circle} = 2\pi r$$
$$= 2 \times \frac{22}{7} \times 3.5$$
$$= 2 \times 22 \times 0.5$$
$$= 22 \text{ m}$$

131]

Example 3.6

A wire of length 88 cm is bent as a circle. What is the radius of the circle.

Solution

$$\text{Length of the wire} = 88 \text{ cm}$$
$$\text{Circumference of the circle} = \text{Length of the wire}$$
$$2\pi r = 88$$
$$2 \times \frac{22}{7} \times r = 88$$
$$r = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

$\therefore$ radius of a circle is 14 cm.

132]

Example 3.8

A scooter wheel makes 50 revolutions to cover a distance of 8800 cm. Find the radius of the wheel.

Solution

$$\text{Distance travelled} = \text{Number of revolutions} \times \text{Circumference}$$

$$\text{Circumference} = \frac{\text{Distance travelled}}{\text{Number of revolutions}}$$

$$2\pi r = \frac{8800}{50}$$

$$\text{i.e., } 2\pi r = 176$$

$$2 \times \frac{22}{7} \times r = 176$$

$$r = \frac{176 \times 7}{2 \times 22}$$

$$r = 28 \text{ cm}$$

$\therefore$ radius of the wheel = 28 cm.

133]

Example 3.13

Find the area of a circle whose diameter is 14 cm

Solution

$$\text{Diameter } d = 14 \text{ cm}$$

So, $$\text{radius } r = \frac{d}{2} = \frac{14}{2} = 7 \text{ cm}$$

$$\text{Area of circle} = \pi r^2$$

$$= \frac{22}{7} \times 7 \times 7$$

$$= 154 \text{ sq. cm}$$

$$\therefore \text{Area of circle} = 154 \text{ sq. cm}$$

134]

Example 3.24

The adjoining figure shows two concentric circles. The radius of the larger circle is 14 cm and the smaller circle is 7 cm. Find

(i) The area of the larger circle.

(ii) The area of the smaller circle.

(iii) The area of the shaded region between two circles.

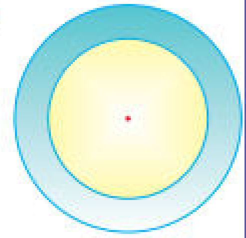


Fig. 3.23

Solution

i) Larger circle

$$R = 14$$
$$\text{area} = \pi R^2 = \frac{22}{7} \times 14 \times 14 = 22 \times 28 = 616 \text{ cm}^2$$

ii) Smaller circle

$$r = 7$$
$$\text{area} = \pi r^2 = \frac{22}{7} \times 7 \times 7 = 22 \times 7 = 154 \text{ cm}^2$$

iii) The area of the shaded region

$$= (\text{Area of larger circle}) - (\text{Area of smaller circle})$$
$$= (616 - 154) \text{ cm}^2 = 462 \text{ cm}^2$$

135]

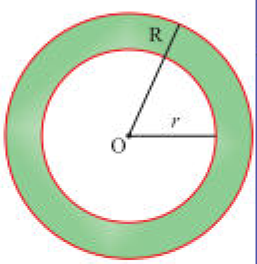


Fig. 3.22

Fig. 3.22

width of the pathway, $w = R - r$ units

i.e., $w = R - r \Rightarrow R = w + r$ units

$r = R - w$ units.

The area of the circular path $=$ (area of the outer circle) $-$ (area of the inner circle)

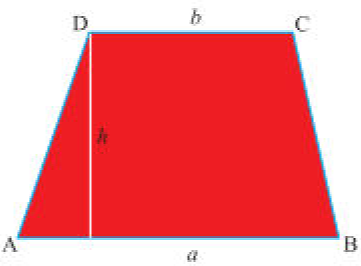
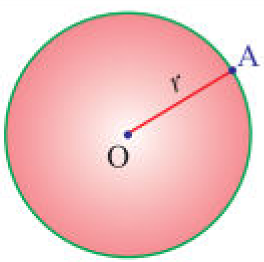
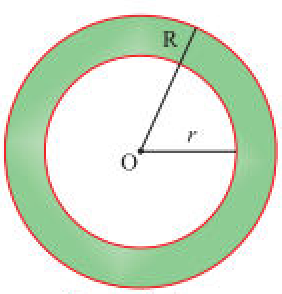
$= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$ sq. units

$\therefore$ The area of the circular path $= \pi(R^2 - r^2)$ sq. units

$= \pi(R + r)(R - r)$ sq. units

136]

Figure	Area	Forumula
 Trapezium	$\frac{1}{2}$ × height × sum of parallel sides	$\frac{1}{2} \times h \times (a + b)$ sq. units
 Circle	Perimeter of the circle = 2 × π × radius	$2\pi r$ units
	Area of the circle = π × radius × radius	πr^2 sq. units
 Circular Pathway	ii) area of the circular pathway	Area of outer circle – Area of inner circle $= \pi\ (R^2 - r^2)$ sq. units $= \pi\ (R + r)\ (R - r)$ sq. units

137]

Example 4.6

Three angles of a triangle are $3x + 5°$, $x + 20°$, $x + 25°$. Find the measure of each angle.

Solution

$$
\begin{aligned}
\text{Sum of the three angles of a triangle} &= 180° \\
3x + 5° + x + 20° + x + 25° &= 180° \\
5x + 50° &= 180° \\
5x &= 180° - 50° \\
5x &= 130° \\
x &= \frac{130°}{5} \\
&= 26° \\
3x + 5° &= (3 \times 26°) + 5° = 78° + 5° = 83° \\
x + 20° &= 26° + 20° = 46° \\
x + 25° &= 26° + 25° = 51°
\end{aligned}
$$

$\therefore$ The three angles of a triangle are 83°, 46° and 51°.

138]

Example 6.4

Find the median of the following data.

3, 4, 5, 3, 6, 7, 2.

Solution:

Arrange the data in ascending order.

2, 3, 3, 4, 5, 6, 7

The number of observation is 7 which is odd.

$\therefore$ The middle value 4 is the median.

139]

Median is defined as the middle value of the data when the data is arranged in ascending or descending order.

Try these

Find the actual distance between your school and house. Find the median of the place.

Find the median of the following:

40, 50, 30, 60, 80, 70

Arrange the given data in ascending order.

30, 40, 50, 60, 70, 80.

Here the number of terms is 6 which is even. So the third and fourth terms are middle terms. The average value of the these terms is the median.

(i.e) Median $= \frac{50+60}{2} = \frac{110}{2} = 55.$

(i) **When the number of observations is odd, the middle number is the median.**

(ii) **When the number of observations is even, the median is the average of the two middle numbers.**

Do you know?

140]

Example 6.2

The monthly income of 6 families are ₹ 3500, ₹ 2700, ₹ 3000, ₹ 2800, ₹ 3900 and ₹ 2100. Find the mean income.

Solution:

$$\text{Average monthly income} = \frac{\text{Total income of 6 familes}}{\text{Number of families}}$$

$$= \frac{₹\ 3500 + 2700 + 3000 + 2800 + 3900 + 2100}{6}$$

$$= ₹\ \frac{18000}{6}$$

$$= ₹\ 3{,}000.$$

Example 6.3

The mean price of 5 pens is ₹ 75. What is the total cost of 5 pens?

Solution:

$$\text{Mean} = \frac{\text{Total cost of 5 pens}}{\text{Number of pens}}$$

$$\text{Total cost of 5 pens} = \text{Mean} \times \text{Number of pens}$$

$$= ₹\ 75 \times 5$$

$$= ₹\ 375$$

141]

Mode

Look at the following example,

Mr. Raghavan, the owner of a ready made dress shop says that the most popular size of shirts he sells is of size 40 cm.

Observe that here also, the owner is concerned about the number of shirts of different sizes sold. He is looking at the shirt size that is sold, the most. The highest occurring event is the sale of size 40 cm. This value is called the mode of the data.

Mode is the variable which occurs most frequently in the given data.

Mode of Large data

Putting the same observation together and counting them is not easy if the number of observation is large. In such cases we tabulate the data.

142]

Example 6.8

Find the mode of the following data.

3, 4, 5, 3, 6, 7

Solution:

3 occurs the most number of times.

∴ Mode of the data is 3.

Try these

Find the mode of the transport in your place.

Example 6.9

Find the mode of the following data.

2, 2, 2, 3, 3, 4, 5,5, 5, 6,6, 8

Solution:

2 and 5 occur 3 times.

∴ Mode of the data is 2 and 5.

Try these

Find the mode of the flower.

Example 6.10

Find the mode of the following data

90, 40, 68, 94, 50, 60.

Solution:

Here there are no frequently occurring values. Hence this data has no mode.

Samacheer Kalvi Maths

8th Std

1.3 Four Properties of Rational Numbers

1.3.1 (a) Addition

(i) Closure property

The sum of any two rational numbers is always a rational number. This is called 'Closure property of addition' of rational numbers. Thus, Q is closed under addition.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b}+\frac{c}{d}$ is also a rational number.

Illustration: (i) $\frac{2}{9}+\frac{4}{9}=\frac{6}{9}=\frac{2}{3}$ is a rational number.

(ii) $5+\frac{1}{3}=\frac{5}{1}+\frac{1}{3}=\frac{15+1}{3}=\frac{16}{3}=5\frac{1}{3}$ is a rational number.

(ii) Commutative property

Addition of two rational numbers is commutative.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b}$.

Illustration: For two rational numbers $\frac{1}{2}, \frac{2}{5}$ we have

$$\frac{1}{2}+\frac{2}{5} = \frac{2}{5}+\frac{1}{2}$$

LHS	RHS
LHS $=\frac{1}{2}+\frac{2}{5}$	RHS $=\frac{2}{5}+\frac{1}{2}$
$=\frac{5+4}{10}=\frac{9}{10}$	$=\frac{4+5}{10}=\frac{9}{10}$

$\therefore$ LHS $=$ RHS

$\therefore$ Commutative property is true for addition.

(iii) Associative property

Addition of rational numbers is associative.

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}$.

Illustration: For three rational numbers $\frac{2}{3}, \frac{1}{2}$ and 2, we have

$$\frac{2}{3}+\left(\frac{1}{2}+2\right) = \left(\frac{2}{3}+\frac{1}{2}\right)+2$$

$$\text{LHS} = \frac{2}{3}+\left(\frac{1}{2}+2\right) = \frac{2}{3}+\left(\frac{1}{2}+\frac{2}{1}\right) = \frac{2}{3}+\left(\frac{1}{2}+\frac{4}{2}\right) = \frac{2}{3}+\frac{5}{2} = \frac{4+15}{6} = \frac{19}{6} = 3\frac{1}{6}$$

$$\text{RHS} = \left(\frac{2}{3}+\frac{1}{2}\right)+2 = \left(\frac{4}{6}+\frac{3}{6}\right)+2 = \frac{7}{6}+2 = \frac{7}{6}+\frac{2}{1} = \frac{7+12}{6} = \frac{19}{6} = 3\frac{1}{6}$$

$\therefore$ LHS = RHS

$\therefore$ Associative property is true for addition.

(iv) Additive identity

The sum of any rational number and zero is the rational number itself.

If $\frac{a}{b}$ is any rational number, then $\frac{a}{b}+0 = \frac{a}{b} = 0+\frac{a}{b}$.

Zero is the additive identity for rational numbers.

Illustration: (i) $\frac{2}{7}+0 = \frac{2}{7} = 0+\frac{2}{7}$

(ii) $\left(\frac{-7}{11}\right)+0 = \frac{-7}{11} = 0+\left(\frac{-7}{11}\right)$

Do you know?

Zero is a special rational number. It can be written as $0 = \frac{0}{q}$ where $q \neq 0$.

(v) Additive inverse

$\left(\frac{-a}{b}\right)$ is the negative or additive inverse of $\frac{a}{b}$.

If $\frac{a}{b}$ is a rational number, then there exists a rational number $\left(\frac{-a}{b}\right)$ such that $\frac{a}{b}+\left(\frac{-a}{b}\right) = 0$.

Illustration: (i) Additive inverse of $\frac{3}{5}$ is $\frac{-3}{5}$

(ii) Additive inverse of $\frac{-3}{5}$ is $\frac{3}{5}$

(iii) Additive inverse of 0 is 0 itself.

1.3.1 (b) Subtraction

(i) Closure Property

The difference between any two rational numbers is always a rational number. Hence Q is closed under subtraction.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} - \frac{c}{d}$ is also a rational number.

Illustration: (i) $\frac{4}{7} - \frac{2}{7} = \frac{2}{7}$ is a rational number.

(ii) $1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$ is a rational number.

(ii) Commutative Property

Subtraction of two rational numbers is not commutative.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} - \frac{c}{d} \neq \frac{c}{d} - \frac{a}{b}$.

Illustration: For two rational numbers $\frac{4}{9}$ and $\frac{2}{5}$, we have

$$\frac{4}{9} - \frac{2}{5} \neq \frac{2}{5} - \frac{4}{9}$$

$$\text{LHS} = \frac{4}{9} - \frac{2}{5} = \frac{20-18}{45} = \frac{2}{45}$$

$$\text{RHS} = \frac{2}{5} - \frac{4}{9} = \frac{18-20}{45} = \frac{-2}{45}$$

$$\therefore \text{LHS} \neq \text{RHS}$$

$\therefore$ Commutative property is not true for subtraction.

Do you know?

When two rational numbers are equal, then commutative property is true for them.

(iii) Associative property

Subtraction of rational numbers is not associative.

If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b}-\left(\frac{c}{d}-\frac{e}{f}\right)\neq\left(\frac{a}{b}-\frac{c}{d}\right)-\frac{e}{f}$.

Illustration: For three rational numbers $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, we have

$$\frac{1}{2}-\left(\frac{1}{3}-\frac{1}{4}\right)\neq\left(\frac{1}{2}-\frac{1}{3}\right)-\frac{1}{4}$$

$$\text{LHS} = \frac{1}{2}-\left(\frac{1}{3}-\frac{1}{4}\right) = \frac{1}{2}-\left(\frac{4-3}{12}\right) = \frac{1}{2}-\left(\frac{1}{12}\right) = \frac{6-1}{12} = \frac{5}{12}$$

$$\text{RHS} = \left(\frac{1}{2}-\frac{1}{3}\right)-\frac{1}{4} = \left(\frac{3-2}{6}\right)-\frac{1}{4} = \frac{1}{6}-\frac{1}{4} = \frac{2-3}{12} = \frac{-1}{12}$$

$$\therefore \text{LHS} \neq \text{RHS}$$

$\therefore$ Associative property is not true for subtraction.

1.3.1 (c) Multiplication

(i) Closure property

The product of two rational numbers is always a rational number. Hence Q is closed under multiplication.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$ is also a rational number.

Illustration: (i) $\frac{1}{3}\times 7=\frac{7}{3}=2\frac{1}{3}$ is a rational number.

(ii) $\frac{4}{3}\times\frac{5}{9}=\frac{20}{27}$ is a rational number.

(ii) Commutative property

Multiplication of rational numbers is commutative.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b}\times\frac{c}{d}=\frac{c}{d}\times\frac{a}{b}$.

Illustration: For two rational numbers $\frac{3}{5}$ and $\frac{-8}{11}$, we have

$$\frac{3}{5}\times\left(\frac{-8}{11}\right) = \left(\frac{-8}{11}\right)\times\frac{3}{5}$$

$$\text{LHS} = \frac{3}{5}\times\left(\frac{-8}{11}\right) = \frac{-24}{55} \qquad \text{RHS} = \frac{-8}{11}\times\left(\frac{3}{5}\right) = \frac{-24}{55}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Commutative property is true for multiplication.

(iii) Associative property

Multiplication of rational numbers is associative.

If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right) = \left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}$.

Illustration: For three rational numbers $\frac{1}{2}$, $\left(\frac{-1}{4}\right)$ and $\frac{1}{3}$, we have

$$\frac{1}{2}\times\left(\frac{-1}{4}\times\frac{1}{3}\right) = \left(\frac{1}{2}\times\left(\frac{-1}{4}\right)\right)\times\frac{1}{3}$$

$$\text{LHS} = \frac{1}{2}\times\left(\frac{-1}{12}\right) = \frac{-1}{24} \qquad \text{RHS} = \left(\frac{-1}{8}\right)\times\frac{1}{3} = \frac{-1}{24}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Associative property is true for multiplication.

(iv) Multiplicative identity

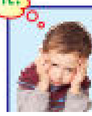
The product of any rational number and 1 is the rational number itself. 'One' is the multiplicative identity for rational numbers.

If $\frac{a}{b}$ is any rational number, then $\frac{a}{b}\times 1 = \frac{a}{b} = 1\times\frac{a}{b}$.

Illustration: (i) $\frac{5}{7}\times 1 = \frac{5}{7}$

(ii) $\left(\frac{-3}{8}\right)\times 1 = \frac{-3}{8}$

Think it!



Is 1 the multiplicative identity for integers?

(v) Multiplication by 0

Every rational number multiplied with 0 gives 0.

If $\frac{a}{b}$ is any rational number, then $\frac{a}{b} \times 0 = 0 = 0 \times \frac{a}{b}$.

Illustration: (i) $-5 \times 0 = 0$

(ii) $\left(\frac{-7}{11}\right) \times 0 = 0$

(vi) Multiplicative Inverse or Reciprocal

For every rational number $\frac{a}{b}$, $a \neq 0$, there exists a rational number $\frac{c}{d}$ such that $\frac{a}{b} \times \frac{c}{d} = 1$. Then $\frac{c}{d}$ is called the multiplicative inverse of $\frac{a}{b}$.

If $\frac{a}{b}$ is a rational number, then $\frac{b}{a}$ is the multiplicative inverse or reciprocal of it.

Illustration: (i) The reciprocal of 2 is $\frac{1}{2}$.

(ii) The multiplicative inverse of $\left(\frac{-3}{5}\right)$ is $\left(\frac{-5}{3}\right)$.

1.3.1 (d) Division

(i) Closure property

The collection of non-zero rational numbers is closed under division.

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, such that $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d}$ is always a rational number.

Illustration: (i) $\frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times \frac{3}{1} = \frac{2}{1} = 2$ is a rational number.

(ii) $\frac{4}{5} \div \frac{3}{2} = \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$ is a rational number.

(ii) Commutative property

Division of rational numbers is not commutative.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$

Illustration: For two rational numbers $\frac{4}{5}$ and $\frac{3}{8}$, we have

$$\frac{4}{5} \div \frac{3}{8} \neq \frac{3}{8} \div \frac{4}{5}$$

$$\text{LHS} = \frac{4}{5} \times \frac{8}{3} = \frac{32}{15} \quad \Big| \quad \text{RHS} = \frac{3}{8} \times \frac{5}{4} = \frac{15}{32}$$

$$\therefore \text{LHS} \neq \text{RHS}$$

$\therefore$ Commutative property is not true for division.

(iii) Associative property

Division of rational numbers is not associative.

If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b} \div \left(\frac{c}{d} \div \frac{e}{f}\right) \neq \left(\frac{a}{b} \div \frac{c}{d}\right) \div \frac{e}{f}$.

Illustration: For three rational numbers $\frac{3}{4}$, 5 and $\frac{1}{2}$, we have

$$\frac{3}{4} \div \left(5 \div \frac{1}{2}\right) \neq \left(\frac{3}{4} \div 5\right) \div \frac{1}{2}$$

$\text{LHS} = \frac{3}{4} \div \left(5 \div \frac{1}{2}\right)$	$\text{RHS} = \left(\frac{3}{4} \div 5\right) \div \frac{1}{2}$
$= \frac{3}{4} \div \left(\frac{5}{1} \times \frac{2}{1}\right)$	$= \left(\frac{3}{4} \times \frac{1}{5}\right) \div \frac{1}{2}$
$= \frac{3}{4} \div 10$	$= \frac{3}{20} \times \frac{2}{1}$
$= \frac{3}{4} \times \frac{1}{10} = \frac{3}{40}$	$= \frac{3}{10}$

$\therefore$ LHS $\neq$ RHS

$\therefore$ Associative property is not true for division.

1.3.1 (e) Distributive Property

(i) Distributive property of multiplication over addition

Multiplication of rational numbers is distributive over addition.

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}$.

Illustration: For three rational numbers $\frac{2}{3}, \frac{4}{9}$ and $\frac{3}{5}$, we have

$$\frac{2}{3} \times \left(\frac{4}{9} + \frac{3}{5}\right) = \frac{2}{3} \times \frac{4}{9} + \frac{2}{3} \times \frac{3}{5}$$

$\text{LHS} = \frac{2}{3} \times \left(\frac{4}{9} + \frac{3}{5}\right)$	$\text{RHS} = \frac{2}{3} \times \frac{4}{9} + \frac{2}{3} \times \frac{3}{5}$
$= \frac{2}{3} \times \left(\frac{20+27}{45}\right)$	$= \frac{8}{27} + \frac{2}{5}$
$= \frac{2}{3} \times \frac{47}{45} = \frac{94}{135}$	$= \frac{40+54}{135} = \frac{94}{135}$

$\therefore$ LHS $=$ RHS

$\therefore$ Multiplication is distributive over addition.

(ii) Distributive property of multiplication over subtraction

Multiplication of rational numbers is distributive over subtraction.

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b} \times \left(\frac{c}{d} - \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} - \frac{a}{b} \times \frac{e}{f}$.

Illustration: For three rational numbers $\frac{3}{7}$, $\frac{4}{5}$ and $\frac{1}{2}$, we have

$$\frac{3}{7}\times\left(\frac{4}{5}-\frac{1}{2}\right) = \frac{3}{7}\times\frac{4}{5}-\frac{3}{7}\times\frac{1}{2}$$

$\text{LHS} = \frac{3}{7}\times\left(\frac{4}{5}-\frac{1}{2}\right)$	$\text{RHS} = \frac{3}{7}\times\frac{4}{5}-\frac{3}{7}\times\frac{1}{2}$
$= \frac{3}{7}\times\left(\frac{8-5}{10}\right)$	$= \frac{12}{35}-\frac{3}{14}$
$= \frac{3}{7}\times\frac{3}{10} = \frac{9}{70}$	$= \frac{24-15}{70} = \frac{9}{70}$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Multiplication is distributive over subtraction.

B - brackets, **O** - of, **D** - division, **M** - multiplication, **A** - addition, **S** - subtraction.

Now we will study more about brackets and operation - of.

Brackets

Some grouping symbols are employed to indicate a preference in the order of operations. Most commonly used grouping symbols are given below.

Grouping symbols	Names
——	Bar bracket or Vinculum
( )	Parenthesis or common brackets
{ }	Braces or Curly brackets
[ ]	Brackets or Square brackets

Operation - "Of "

We sometimes come across expressions like ‘twice of 3’, ‘one - fourth of 20’, ‘half of 10’ etc. In these expressions, ‘of’ means ‘multiplication with’.

For example,

(i) ‘twice of 3’ is written as 2×3,

(ii) ‘one - fourth’ of 20 is written as $\frac{1}{4}\times 20$,

(iii) ‘half of 10’ is written as $\frac{1}{2}\times 10$.

If more than one grouping symbols are used, we first perform the operations within the innermost symbol and remove it. Next we proceed to the operations within the next innermost symbols and so on.

1.6 Laws of Exponents with Integral Powers

With the above definition of positive integral power of a real number, we now establish the following properties called "laws of indices" or "laws of exponents".

(i) Product Rule

Law 1	$a^m \times a^n = a^{m+n}$, where 'a' is a real number and m, n are positive integers

Illustration

$\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{3+4} = \left(\frac{2}{3}\right)^7$ (Using the law, $a^m \times a^n = a^{m+n}$, where $a = \frac{2}{3}$, $m = 3$, $n = 4$)

(ii) Quotient Rule

Law 2	$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$ and m, n are positive integers with $m > n$

Illustration

$\frac{6^4}{6^2} = 6^{4-2} = 6^2$ **(Using the law $\frac{a^m}{a^n} = a^{m-n}$, where a = 6, m=4, n=2)**

(iii) Power Rule

Law 3	$(a^m)^n = a^{m \times n}$, where m and n are positive integers

Illustration

$(3^2)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2 = 3^{2+2+2+2} = 3^8$

we can get the same result by multiplying the two powers

i.e, $(3^2)^4 = 3^{2\times4} = 3^8$.

Try these

Show that $a^{(x-y)z} \times a^{(y-z)x} \times a^{(z-x)y} = 1$

(iv) Number with zero exponent

For $m \neq o$,

$m^3 \div m^3 = m^{3-3} = m^0$ (using law 2);

Aliter:

$m^3 \div m^3 = \frac{m^3}{m^3} = \frac{m \times m \times m}{m \times m \times m} = 1$

Using these two methods, $m^3 \div m^3 = m^0 = 1$.

From the above example, we come to the **fourth law** of exponent

Law 4	If 'a' is a rational number other than "zero", then $a^0 = 1$

Illustration

(i) $2^0 = 1$ (ii) $\left(\frac{3}{4}\right)^0 = 1$ (iii) $25^0 = 1$ (iv) $\left(-\frac{2}{5}\right)^0 = 1$ (v) $(-100)^0 = 1$

(v) Law of Reciprocal

The value of a number with negative exponent is calculated by converting into multiplicative inverse of the same number with positive exponent.

Illustration

(i) $4^{-4} = \frac{1}{4^4} = \frac{1}{4 \times 4 \times 4 \times 4} = \frac{1}{256}$

(ii) $5^{-3} = \frac{1}{5^3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125}$

(iii) $10^{-2} = \frac{1}{10^2} = \frac{1}{10 \times 10} = \frac{1}{100}$

Reciprocal of 3 is equal to $\frac{1}{3} = \frac{3^0}{3^1} = 3^{0-1} = 3^{-1}$.

Similarly, reciprocal of $6^2 = \frac{1}{6^2} = \frac{6^0}{6^2} = 6^{0-2} = 6^{-2}$

Further, reciprocal of $\left(\frac{8}{3}\right)^3$ is equal to $\frac{1}{\left(\frac{8}{3}\right)^3} = \left(\frac{8}{3}\right)^{-3}$.

From the above examples, we come to the fifth law of exponent.

Law 5	If 'a' is a real number and 'm' is an integer, then $a^{-m} = \frac{1}{a^m}$

(vi) Multiplying numbers with same exponents

Consider the simplifications,

(i) $4^3 \times 7^3 = (4 \times 4 \times 4) \times (7 \times 7 \times 7) = (4 \times 7) \times (4 \times 7) \times (4 \times 7)$

$= (4 \times 7)^3$

(ii) $5^{-3} \times 4^{-3} = \frac{1}{5^3} \times \frac{1}{4^3} = \left(\frac{1}{5}\right)^3 \times \left(\frac{1}{4}\right)^3$

$= \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

$= \left(\frac{1}{5} \times \frac{1}{4}\right) \times \left(\frac{1}{5} \times \frac{1}{4}\right) \times \left(\frac{1}{5} \times \frac{1}{4}\right) = \left(\frac{1}{20}\right)^3$

$= 20^{-3} = (5 \times 4)^{-3}$

(iii) $\left(\frac{3}{5}\right)^2 \times \left(\frac{1}{2}\right)^2 = \left(\frac{3}{5} \times \frac{3}{5}\right) \times \left(\frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{5} \times \frac{1}{2}\right) \times \left(\frac{3}{5} \times \frac{1}{2}\right)$

$= \left(\frac{3}{5} \times \frac{1}{2}\right)^2$

In general, for any two integers a and b we have

$$a^2 \times b^2 = (a \times b)^2 = (ab)^2$$

$\therefore$ We arrive at the **power of a product rule** as follows:

$(a \times a \times a \times \ldots m \text{ times}) \times (b \times b \times b \times \ldots m \text{ times}) = ab \times ab \times ab \times \ldots m \text{ times} = (ab)^m$

(i.e.,) $a^m \times b^m = (ab)^m$

Law 6	$a^m \times b^m = (ab)^m$, where a, b are real numbers and m is an integer.

Illustration

(i) $3^x \times 4^x = (3 \times 4)^x = 12^x$

(ii) $7^2 \times 2^2 = (7 \times 2)^2 = 14^2 = 196$

(vii) Power of a quotient rule

Consider the simplifications,

(i) $\left(\frac{4}{3}\right)^2 = \frac{4}{3} \times \frac{4}{3} = \frac{16}{9} = \frac{4^2}{3^2}$ and

(ii) $\left(\frac{3}{5}\right)^{-2} = \frac{1}{\left(\frac{3}{5}\right)^2} = \frac{1}{\left(\frac{3^2}{5^2}\right)} = \frac{5^2}{3^2} = \left(\frac{5}{3}\right)^2 \quad \left(\because a^{-m} = \frac{1}{a^m}\right)$

$= \frac{5}{3} \times \frac{5}{3} = \frac{5 \times 5}{3 \times 3} = \frac{5^2}{3^2} = 5^2 \times \frac{1}{3^2} = 5^2 \times 3^{-2} = \frac{1}{5^{-2}} \times 3^{-2}$

$= \frac{3^{-2}}{5^{-2}}.$

Hence $\left(\frac{a}{b}\right)^2$ can be written as $\frac{a^2}{b^2}$

$$\left(\frac{a}{b}\right)^m = \left(\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \ldots m \text{ times}\right) = \frac{a \times a \times a \ldots m \text{ times}}{b \times b \times b \times \ldots m \text{ times}}$$

$$\therefore \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Law 7	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$, a and b are real numbers, m is an integer

Illustration

(i) $\left(\frac{a}{b}\right)^7 = \frac{a^7}{b^7}$ (ii) $\left(\frac{5}{3}\right)^3 = \frac{5^3}{3^3} = \frac{125}{27}$

(iii) $\left(\frac{1}{4}\right)^4 = \frac{1^4}{4^4} = \frac{1}{256}$

Example 1.8

Simplify: (i) $2^5 \times 2^3$ (ii) $10^9 \div 10^6$ (iii) $(x^0)^4$ (iv) $(2^3)^0$

(v) $\left(\frac{3}{2}\right)^5$ (vi) $(2^5)^2$ (vii) $(2 \times 3)^4$

(viii) If $2^p = 32$, find the value of p.

Solution

(i) $2^5 \times 2^3 = 2^{5+3} = 2^8$

(ii) $10^9 \div 10^6 = 10^{9-6} = 10^3$

(iii) $(x^0)^4 = (1)^4 = 1$ $[\because a^0 = 1]$

(iv) $(2^3)^0 = 8^0 = 1$ $[\because a^0 = 1]$

(v) $\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5} = \frac{243}{32}$

(vi) $(2^5)^2 = 2^{5\times 2} = 2^{10} = 1024$

(vii) $(2 \times 3)^4 = 6^4 = 1296$

(or) $(2 \times 3)^4 = 2^4 \times 3^4 = 16 \times 81 = 1296$

(viii) Given : $2^p = 32$

$2^p = 2^5$

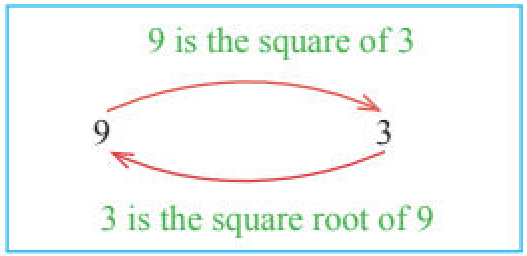
Therefore $p = 5$ (Here the base on both sides are equal.)

2	32
2	16
2	8
2	4
2	2
	1

1.7.2 Square roots

Definition

When a number is multiplied by itself, the product is called the square of that number. The number itself is called the **square root** of the product.



For example:

(i) $3 \times 3 = 3^2 = 9$

(ii) $(-3) \times (-3) = (-3)^2 = 9$

Here 3 and (-3) are the square roots of 9.

The symbol used for square root is $\sqrt{\ }$.

$\therefore \sqrt{9} = \pm 3$ (read as plus or minus 3)

Considering only the positive root, we have $\sqrt{9} = 3$

Note: We write the square root of x as $\sqrt{x}$ or $x^{\frac{1}{2}}$.

Hence, $\sqrt{4} = (4)^{\frac{1}{2}}$ and $\sqrt{100} = (100)^{\frac{1}{2}}$

To find a square root of a number, we have the following two methods.

(i) Factorization Method

(ii) Long Division Method

(i) Factorization Method

The square root of a perfect square number can be found by finding the prime factors of the number and grouping them in pairs.

Example 1.17

Find the square root of 64

Solution

Prime factorization

2	64
2	32
2	16
2	8
2	4
2	2
	1

$$64 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} = 2^2 \times 2^2 \times 2^2$$

$$\sqrt{64} = \sqrt{2^2 \times 2^2 \times 2^2} = 2 \times 2 \times 2 = 8$$

$$\sqrt{64} = 8$$

(ii) Long division method

In case of large numbers, factors can not be found easily. Hence we may use another method, known as **Long division method**.

Using this method, we can also find square roots of decimal numbers. This method is explained in the following worked examples.

Example 1.23

Find the square root of 529 using long division method.

Solution

Step 1 : We write 529 as $5\ \overline{29}$ by grouping the numbers in pairs, starting from the right end. (i.e. from the unit's place).

Step 2 : Find the number whose square is less than (or equal to) 5. Here it is 2.

$$\begin{array}{r|l} 2 & 5\ \overline{29} \end{array}$$

Step 3 : Put '2' on the top, and also write 2 as a divisor as shown.

$$\begin{array}{r|l} & 2 \\ \hline 2 & 5\ \overline{29} \\ & 4 \\ \hline & 1 \end{array}$$

Step 4 : Multiply 2 on the top with the divisor 2 and write 4 under 5 and subtract. The remainder is 1.

Step 5 : Bring down the pair 29 by the side of the remainder 1, yielding 129.

$$\begin{array}{r|l} & 2 \\ \hline 2 & 5\ \overline{29} \\ & 4\ \downarrow \\ \hline & 1\ 29 \end{array}$$

Step 6 : Double 2 and take the resulting number 4. Find that number 'n' such that $4n \times n$ is just less than or equal to 129.

For example : $42 \times 2 = 84$; and $43 \times 3 = 129$ and so $n = 3$.

$$\begin{array}{r|l} & 2\ \ 3 \\ \hline 2 & 5\ \overline{29} \\ & 4\ \downarrow \\ \hline 43 & 1\ 29 \\ & 1\ 29 \\ \hline & \quad 0 \end{array}$$

Step 7 : Write 43 as the next divisor and put 3 on the top along with 2. Write the product $43 \times 3 = 129$ under 129 and subtract. Since the remainder is '0', the division is complete.

Hence $\sqrt{529} = 23$.

Example 1.25

Find the square root of 6.0516

Solution

We write the number as $6\,.\,\overline{05}\,\overline{16}$. Since the number of digits in the integral part is 1, the square root will have 1 digit in its integral part. We follow the same procedure that we usually use to find the square root of 60516

$$
\begin{array}{r|l}
 & \;\;2.\,4\;\;\,6 \\
\hline
2 & 6.\overline{05}\,\overline{16} \\
 & 4 \\
\hline
44 & 2\;05 \\
 & 1\;76 \\
\hline
486 & \;\;\;29\,16 \\
 & \;\;\;29\,16 \\
\hline
 & \;\;\;\;\;\;0
\end{array}
$$

From the above working, we get $\sqrt{6.0516} = 2.46$.

Example 1.31

Find the square root of 3 correct to two places of decimal.

Solution

$$
\begin{array}{r|l}
 & 1.\;\;7\;\;\;3\;\;\;2 \\
\hline
1 & 3.\,\overline{00}\,\overline{00}\,\overline{00} \\
 & 1 \\
\hline
27 & 2\;\;00 \\
 & 1\;\;89 \\
\hline
343 & \;\;\;11\,00 \\
 & \;\;\;10\,29 \\
\hline
3462 & \;\;\;\;\;\;71\;\;00 \\
 & \;\;\;\;\;\;69\;\;24 \\
\hline
 & \;\;\;\;\;\;\;\;1\;\;76
\end{array}
$$

> Since we need the answer correct to two places of decimal, we shall first find the square root up to three places of decimal. For this purpose we must add 6 (that is three pairs of) zeros to the right of the decimal point.

$\therefore \sqrt{3} = 1.732$ up to three places of decimal.

$\sqrt{3} = 1.73$ correct to two places of decimal.

Example 1.32

Find the square root of $10\frac{2}{3}$ correct to two places of decimal.

Solution

$10\frac{2}{3} = \frac{32}{3} = 10.66\ 66\ 66$ ....

$$\begin{array}{r|l} & \quad 3.\ \ 2\ \ 6\ \ 5 \\ \hline 3 & 10.\ \overline{66}\ \overline{66}\ \overline{67} \\ & \ \ 9 \\ \hline 62 & \ \ 1\ 66 \\ & \ \ 1\ 24 \\ \hline 646 & \quad 42\ 66 \\ & \quad 38\ 76 \\ \hline 6525 & \quad\ \ 3\ 90\ 67 \\ & \quad\ \ 3\ 26\ 25 \\ \hline & \qquad\ 64\ 42 \end{array}$$

In order to find the square root correct to two places of decimal, we have to find the square root up to three places. Therefore we have to convert $\frac{2}{3}$ as a decimal correct to six places.

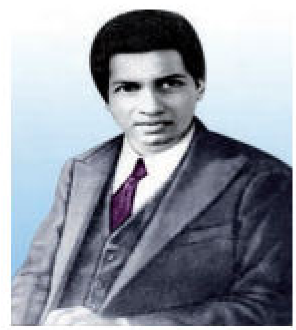
$$\sqrt{10\frac{2}{3}} = 3.265 \text{ (approximately)}$$

$$= 3.27 \text{ (correct to two places of decimal)}$$

1.7.3 Cubes

Introduction

This is an incident about one of the greatest mathematical geniuses S. Ramanujan. Once mathematician Prof. G.H. Hardy came to visit him in a taxi whose taxi number was 1729. While talking to Ramanujan, Hardy described that the number 1729 was a dull number. Ramanujan quickly pointed out that 1729 was indeed an interesting number. He said, it is the smallest number that can be expressed as a sum of two cubes in two different ways.



Srinivasa Ramanujan
(1887 -1920)

Ramanujan, an Indian Mathematician who was born in Erode contributed the theory of numbers which brought him worldwide acclamation. During his short life time, he independently compiled nearly 3900 results.

ie., $1729 = 1728 + 1 = 12^3 + 1^3$

and $1729 = 1000 + 729 = 10^3 + 9^3$

1729 is known as the Ramanujan number.

There are many other interesting patterns of cubes, cube roots and the facts related to them.

Cubes

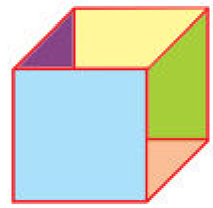
Do you know?

1729 is the smallest Ramanujan Number. There are an infinitely many such numbers. Few are 4104 (2, 16 ; 9, 15), 13832 (18, 20 ; 2, 24).

We know that the word 'Cube' is used in geometry. A cube is a solid figure which has all its sides are equal.

If the side of a cube in the adjoining figure is 'a' units

then its volume is given by $a \times a \times a = a^3$ cubic units.



Here a^3 is called **"a cubed"** or **"a raised to the power three"** or **"a to the power 3"**.

Now, consider the number 1, 8, 27, 64, 125, $\cdots$

These are called **perfect cubes** or **cube numbers**.

Each of them is obtained when a number is multiplied by itself three times.

Examples: $1 \times 1 \times 1 = 1^3$, $2 \times 2 \times 2 = 2^3$, $3 \times 3 \times 3 = 3^3$, $5 \times 5 \times 5 = 5^3$

Example 1.33

Find the value of the following :

(i) 15^3 (ii) $(-4)^3$ (iii) $(1.2)^3$ (iv) $\left(\frac{-3}{4}\right)^3$

Solution

(i) $15^3 = 15 \times 15 \times 15 = 3375$

(ii) $(-4)^3 = (-4) \times (-4) \times (-4) = -64$

(iii) $(1.2)^3 = 1.2 \times 1.2 \times 1.2 = 1.728$

(iv) $\left(\frac{-3}{4}\right)^3 = \frac{(-3) \times (-3) \times (-3)}{4 \times 4 \times 4} = \frac{-27}{64}$

Observe the question (ii) Here $(-4)^3 = -64$.

Note: When a negative number is multiplied by itself an even number of times, the product is positive. But when it is multiplied by itself an odd number of times, the product is also negative. ie, $(-1)^n = \begin{cases} -1 & \text{if n is odd} \\ +1 & \text{if n is even} \end{cases}$

Adding consecutive odd numbers

Observe the following pattern of sums of odd numbers.

$$1 = 1 = 1^3$$

Next 2 odd numbers, $3 + 5 = 8 = 2^3$

Next 3 odd numbers, $7 + 9 + 11 = 27 = 3^3$

Next 4 odd numbers, $13 + 15 + 17 + 19 = 64 = 4^3$

Next 5 odd numbers, $21 + 23 + 25 + 27 + 29 = 125 = 5^3$

Is it not interesting?

Example 1.37

Find the cube root of 512.

Solution

$$\begin{aligned}\sqrt[3]{512} &= (512)^{\frac{1}{3}} \\ &= ((2\times2\times2)\times(2\times2\times2)\times(2\times2\times2))^{\frac{1}{3}} \\ &= (2^3\times2^3\times2^3)^{\frac{1}{3}} \\ &= (2^9)^{\frac{1}{3}} = 2^3 \\ \sqrt[3]{512} &= 8.\end{aligned}$$

Prime factorization

$$\begin{array}{r|l} 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

Example 1.38

Find the cube root of 27×64

Solution

Resolving 27 and 64 into prime factors, we get

$$\sqrt[3]{27} = (3\times3\times3)^{\frac{1}{3}} = (3^3)^{\frac{1}{3}}$$

$$\sqrt[3]{27} = 3$$

$$\begin{aligned}\sqrt[3]{64} &= (2\times2\times2\times2\times2\times2)^{\frac{1}{3}} \\ &= (2^6)^{\frac{1}{3}} = 2^2 = 4 \\ \sqrt[3]{64} &= 4\end{aligned}$$

$$\begin{aligned}\sqrt[3]{27\times64} &= \sqrt[3]{27}\times\sqrt[3]{64} \\ &= 3\times4 \\ \sqrt[3]{27\times64} &= 12\end{aligned}$$

Prime factorization

$$\begin{array}{r|l} 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Prime factorization

$$\begin{array}{r|l} 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

The shape of each of these objects is a 'circle'.

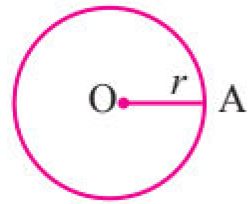


Fig. 2.3

(iii) Circle

Let 'O' be the centre of a circle with radius 'r' units $(\overline{OA})$.

Area of a circle, A = πr^2 sq.units.

Perimeter or circumference of a circle,

$$P = 2\pi r \text{ units,}$$

where $\pi \simeq \frac{22}{7}$ or 3.14.

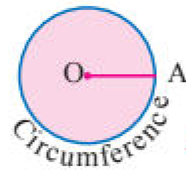


Fig. 2.4

Note: The central angle of a circle is 360°.

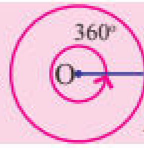


Fig. 2.5

2.2 Semi circles and Quadrants

2.2.1 Semicircle

Have you ever noticed the sky during night time after 7 days of new moon day or full moon day?

What will be the shape of the moon?

It looks like the shape of Fig. 2.6.

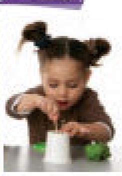
How do you call this?


Fig. 2.6

This is called a semicircle. [Half part of a circle]

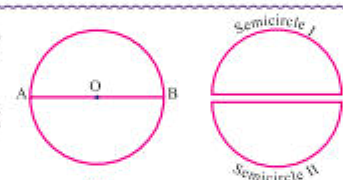
The two equal parts of a circle divided by its diameter are called semicircles.

Activity



How will you get a semicircle from a circle?

Take a cardboard of circular shape and cut it through its diameter $\overline{AB}$.



(a) Fig. 2.7 (b)

Note: The central angle of the semicircle is 180°.

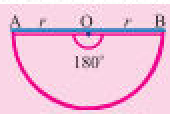


Fig. 2.8

(a) Perimeter of a semicircle

$$\text{Perimeter, P} = \frac{1}{2} \times (\text{circumference of a circle}) + 2 \times r \text{ units}$$
$$= \frac{1}{2} \times 2\pi r + 2r$$
$$\text{P} = \pi r + 2r = (\pi + 2)\, r \text{ units}$$

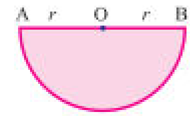


Fig. 2.9

(b) Area of a semicircle

$$\text{Area, A} = \frac{1}{2} \times (\text{Area of a circle})$$
$$= \frac{1}{2} \times \pi r^2$$
$$\text{A} = \frac{\pi r^2}{2} \text{ sq. units.}$$

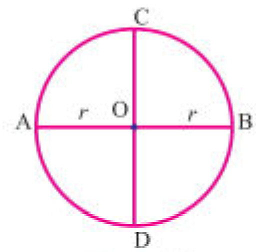


Fig. 2.10

4.2.2 Quadrant of a circle

Cut the circle through two of its perpendicular diameters. We get four equal parts of the circle. Each part is called a quadrant of the circle. We get four quadrants OCA, OAD, ODB and OBC while cutting the circle as shown in the Fig. 2.11.

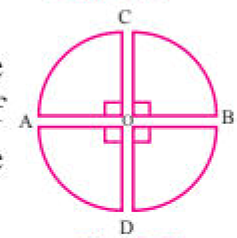


Fig. 2.11

Note: The central angle of the quadrant is 90°.

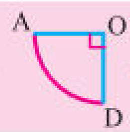


Fig. 2.12

(a) Perimeter of a quadrant

$$\text{Perimeter, P} = \frac{1}{4} \times (\text{circumference of a circle}) + 2r \text{ units}$$
$$= \frac{1}{4} \times 2\pi r + 2r$$
$$\text{P} = \frac{\pi r}{2} + 2r = \left(\frac{\pi}{2} + 2\right) r \text{ units}$$

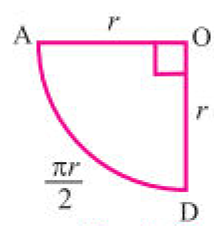
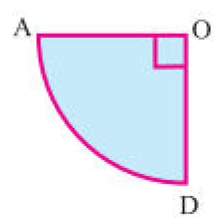


Fig. 2.13

(b) Area of a quadrant

$$\text{Area, A} = \frac{1}{4} \times (\text{Area of a circle})$$
$$\text{A} = \frac{1}{4} \times \pi r^2 \text{ sq.units}$$



Example 2.1

Fig. 2.14

Find the perimeter and area of a semicircle whose radius is 14 cm.

Solution

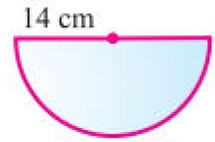


Fig. 2.15

Given: Radius of a semicircle, $r = 14$ cm

Perimeter of a semicircle, $P = (\pi + 2)\, r$ units

$$\therefore P = \left(\frac{22}{7} + 2\right) \times 14$$

$$= \left(\frac{22+14}{7}\right) \times 14 = \frac{36}{7} \times 14 = 72$$

Perimeter of the semicircle $= 72$ cm.

Area of a semicircle, $A = \frac{\pi r^2}{2}$ sq. units

$$\therefore A = \frac{22}{7} \times \frac{14 \times 14}{2} = 308 \text{ cm}^2.$$

Example 2.2

The radius of a circle is 21 cm. Find the perimeter and area of a quadrant of the circle.

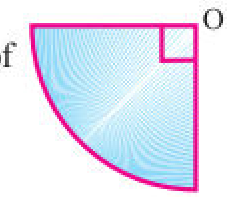


Fig. 2.16

Solution

Given: Radius of a circle, $r = 21$ cm

Perimeter of a quadrant, $P = \left(\frac{\pi}{2} + 2\right) r$ units

$$= \left(\frac{22}{7 \times 2} + 2\right) \times 21 = \left(\frac{22}{14} + 2\right) \times 21$$

$$P = \left(\frac{22+28}{14}\right) \times 21 = \frac{50}{14} \times 21$$

$$= 75 \text{ cm}.$$

Area of a quadrant, $A = \frac{\pi r^2}{4}$ sq. units

$$A = \frac{22}{7} \times \frac{21 \times 21}{4}$$

$$= 346.5 \text{ cm}^2.$$

Example 2.3

The diameter of a semicircular grass plot is 14 m. Find the cost of fencing the plot at ₹ 10 per metre .

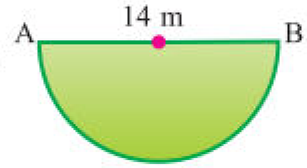


Fig. 2.17

Solution

Given: Diameter, $d = 14$ m.

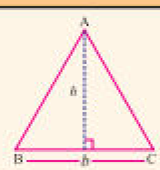
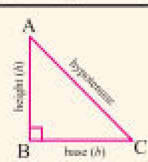
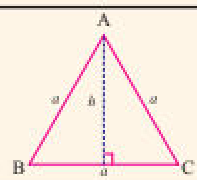
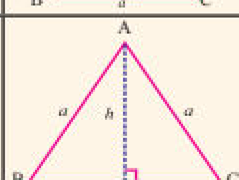
$\therefore$ Radius of the plot, $r = \frac{14}{2} = 7$m.

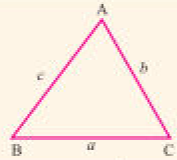
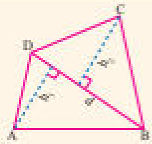
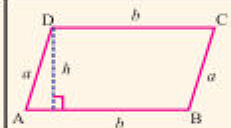
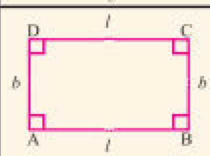
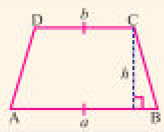
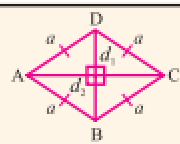
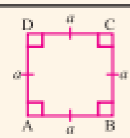
To fence the semicircular plot, we have to find the perimeter of it.

$$\begin{aligned} \text{Perimeter of a semicircle, P} &= (\pi + 2) \times \text{r units} \\ &= \left(\frac{22}{7} + 2\right) \times 7 \\ &= \left(\frac{22 + 14}{7}\right) \times 7 \\ \text{P} &= 36 \text{ m} \end{aligned}$$

Cost of fencing the plot for 1 metre = ₹ 10

$\therefore$ Cost of fencing the plot for 36 metres = 36 × 10 = ₹ 360.

No.	Name of the Figure	Figure	Area (A) (sq. units)	Perimeter (P) (units)
1.	Triangle		$\frac{1}{2} \times b \times h$	AB + BC + CA
2.	Right triangle		$\frac{1}{2} \times b \times h$	(base + height + hypotenuse)
3.	Equilateral triangle		$\frac{\sqrt{3}}{4}a^2$ where ($\sqrt{3} \simeq 1.732$)	AB+BC+CA = $3a$; Altitude, $h = \frac{\sqrt{3}}{2}a$ units
4.	Isosceles triangle		$h \times \sqrt{a^2 - h^2}$	$2a + 2\sqrt{a^2 - h^2}$

5.	Scalene triangle		$\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$	AB + BC + CA $= (a+b+c)$
6.	Quadrilateral		$\frac{1}{2} \times d \times (h_1 + h_2)$	AB + BC + CD + DA
7.	Parallelogram		$b \times h$	$2 \times (a+b)$
8.	Rectangle		$l \times b$	$2 \times (l+b)$
9.	Trapezium		$\frac{1}{2} \times h \times (a+b)$	AB + BC + CD + DA
10.	Rhombus		$\frac{1}{2} \times d_1 \times d_2$ where d_1, d_2 are diagonals	$4a$
11.	Square		a^2	$4a$

Example 2.5

Find the perimeter and area of the following combined figures.

i)

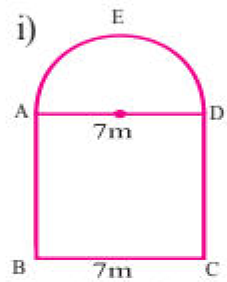


Fig. 2.26

(ii)

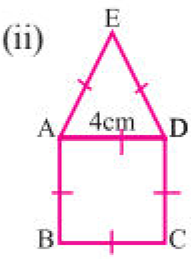
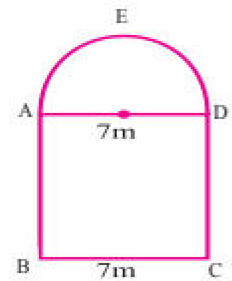


Fig. 2.27

Solution

(i) It is a combined figure made up of a square ABCD and a semicircle DEA. Here, are $\widehat{DEA}$ is half the circumference of a circle whose diameter is AD.



Given: Side of a square = 7 m

$\therefore$ Diameter of a semicircle = 7 m

$\therefore$ Radius of a semicircle, r = $\frac{7}{2}$ m

Perimeter of the combined figure = $\overline{AB}+\overline{BC}+\overline{CD}+\widehat{DEA}$

$$P = 7+7+7+\frac{1}{2}\times \text{(circumference of a circle)}$$

$$= 21+\frac{1}{2}\times 2\pi r = 21+\frac{22}{7}\times\frac{7}{2}$$

$$P = 21+11 = 32 \text{ m}$$

. Perimeter of the combined figure = 32 m.

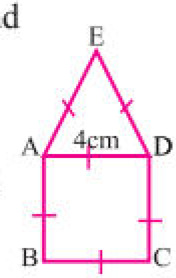
Area of the combined figure = Area of a semicircle + Area of a square

$$A = \frac{\pi r^2}{2}+a^2$$

$$= \frac{22}{7\times 2}\times\frac{7\times 7}{2\times 2}+7^2 = \frac{77}{4}+49$$

$\therefore$ Area of the given combined figure $= 19.25+49 = 68.25 \text{ m}^2$.

(ii) The given combined figure is made up of a square ABCD and an equilateral triangle DEA.



Given: Side of a square $= 4$ cm

$\therefore$ Perimeter of the combined figure $= AB + BC + CD + DE + EA$

$= 4 + 4 + 4 + 4 + 4 = 20$ cm

$\therefore$ Perimeter of the combined figure $= 20$ cm.

Area of the given combined figure $=$ Area of a square + Area of an equilateral triangle

$= a^2 + \frac{\sqrt{3}}{4}a^2$ $\quad \sqrt{3} = 1.732$

$= 4 \times 4 + \frac{\sqrt{3}}{4} \times 4 \times 4$

$= 16 + 1.732 \times 4$

Area of the given combined figure $= 16 + 6.928 = 22.928$

Area of the given figure $\simeq 22.93 \text{ cm}^2$.

Example 2.6

Find the perimeter and area of the shaded portion

(i)

Fig. 2.28

(ii)

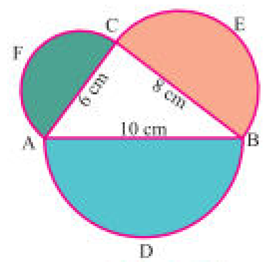
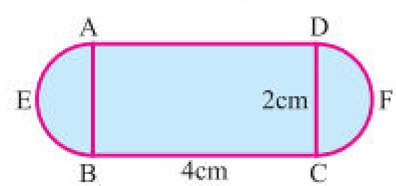


Fig. 2.29

Solution

(i) The given figure is a combination of a rectangle ABCD and two semicircles AEB and DFC of equal area.



Given: Length of the rectangle, $l = 4$ cm

Breadth of the rectangle, $b = 2$ cm

Diameter of a semicircle $= 2$ cm

$\therefore$ Radius of a semicircle, $r = \frac{2}{2} = 1$ cm

$\therefore$ Perimeter of the given figure $= AD + BC + \widehat{AEB} + \widehat{DFC}$

$= 4 + 4 + 2 \times \frac{1}{2} \times$ (circumference of a circle)

$= 4+4+2\times\frac{1}{2}\times$ (circumference of a circle)

$= 8+2\times\frac{1}{2}\times 2\pi r$

$= 8+2\times\frac{22}{7}\times 1$

$= 8+2\times 3.14$

$= 8+6.28$

$\therefore$ Perimeter of the given figure = 14.28 cm.

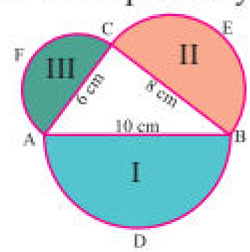
Area of the given figure = Area of a rectangle ABCD + 2 × Area of a semicircle

$= l\times b+2\times\frac{\pi r^2}{2}$

$= 4\times 2+2\times\frac{22\times 1\times 1}{7\times 2}$

$\therefore$ Total area $= 8+3.14 = 11.14$ cm^2.

(ii) Let ADB, BEC and CFA be the three semicircles I, II and III respectively.



Given:

Radius of a semicircle I, $r_1 = \frac{10}{2} = 5$ cm

Radius of a semicircle II, $r_2 = \frac{8}{2} = 4$ cm

Radius of a semicircle III, $r_3 = \frac{6}{2} = 3$ cm

Perimeter of the shaded portion = Perimeter of a semicircle I + Perimeter of a semicircle II + Perimeter of a semicircle III

$$= (\pi + 2)\times 5 + (\pi + 2)\times 4 + (\pi + 2)\times 3$$

$$= (\pi + 2)(5 + 4 + 3) = (\pi + 2)\times 12$$

$$= \left(\frac{22 + 14}{7}\right)\times 12 = \frac{36}{7}\times 12 = 61.714$$

Perimeter of the shaded portion $\simeq$ 61.71cm.

Area of the shaded portion, A = Area of a semicircle I + Area of a semicircle II + Area of a semicircle III

$$A = \frac{\pi r_1^2}{2} + \frac{\pi r_2^2}{2} + \frac{\pi r_3^2}{2}$$

$$= \frac{22}{7\times 2}\times 5\times 5 + \frac{22}{7\times 2}\times 4\times 4 + \frac{22}{7\times 2}\times 3\times 3$$

$$A = \frac{275}{7} + \frac{176}{7} + \frac{99}{7} = \frac{550}{7} = 78.571\text{ cm}^2$$

Area of the shaded portion $\simeq 78.57\text{ cm}^2$

In this example we observe that,

Area of semicircle BEC + Area of semicircle CFA = Area of semicircle ADB

Example 2.7

A horse is tethered to one corner of a rectangular field of dimensions 70 m by 52 m by a rope 28 m long for grazing. How much area can the horse graze inside? How much area is left ungrazed?

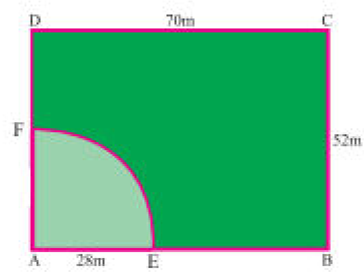


Fig. 2.30

Solution

Length of the rectangle, l = 70 m

Breadth of the rectangle, b = 52 m

Length of the rope = 28 m

Shaded portion AEF indicates the area in which the horse can graze. Clearly, it is the area of a quadrant of a circle of radius, $r = 28$ m

Area of the quadrant AEF = $\frac{1}{4}\times \pi r^2$ sq. units

$$= \frac{1}{4}\times \frac{22}{7}\times 28\times 28 = 616\text{ m}^2$$

$\therefore$ Grazing Area = 616 m^2.

$\therefore$ Grazing Area $= 616\ m^2$.

Area left ungrazed $=$ Area of the rectangle ABCD $-$ Area of the quadrant AEF

Area of the rectangle ABCD $= l \times b$ sq. units

$= 70 \times 52 = 3640\ m^2$

$\therefore$ Area left ungrazed $= 3640 - 616 = 3024\ m^2$.

Example 2.8

In the given figure, ABCD is a square of side 14 cm. Find the area of the shaded portion.

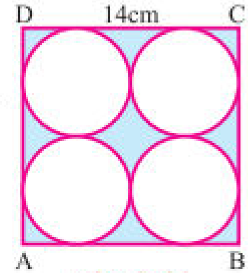


Fig. 2.31

Solution

Side of a square, $a = 14$ cm

Radius of each circle, $r = \frac{7}{2}$ cm

Area of the shaded portion $=$ Area of a square $-\ 4 \times$ Area of a circle

$= a^2 - 4(\pi r^2)$

$= 14 \times 14 - 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 196 - 154$

$\therefore$ Area of the shaded portion $= 42\ cm^2$.

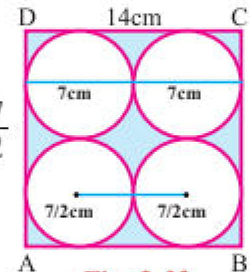


Fig. 2.32

Example 2.9

A copper wire is in the form of a circle with radius 35 cm. It is bent into a square. Determine the side of the square.

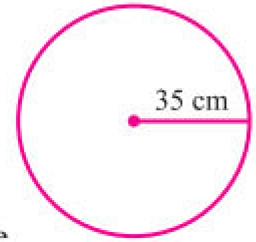


Fig. 2.33

Solution

Given: Radius of a circle, r = 35 cm.

Since the same wire is bent into the form of a square,

Perimeter of the circle = Perimeter of the square

Perimeter of the circle = $2\pi r$ units

$$= 2 \times \frac{22}{7} \times 35 \text{ cm}$$

P = 220 cm.

Let 'a' be the side of a square.

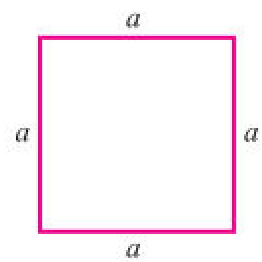


Fig. 2.34

Perimeter of a square = 4a units

$4a = 220$

$a = 55$ cm

$\therefore$ Side of the square = 55 cm.

Example 2.10

Four equal circles are described about four corners of a square so that each touches two of the others as shown in the Fig. 2.35. Find the area of the shaded portion, each side of the square measuring 28 cm.

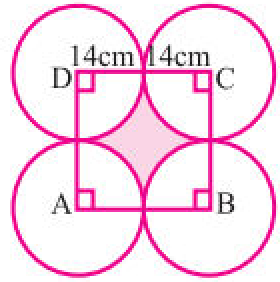


Fig. 2.35

Solution

Let ABCD be the given square of side a.

$$\therefore a = 28\text{cm}$$

$$\therefore \text{Radius of each circle, } r = \frac{28}{2} = 14 \text{ cm}$$

$$\text{Area of the shaded portion} = \text{Area of a square} - 4 \times \text{Area of a quadrant}$$

$$= a^2 - 4 \times \frac{1}{4} \times \pi r^2$$

$$= 28 \times 28 - 4 \times \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$$

$$= 784 - 616$$

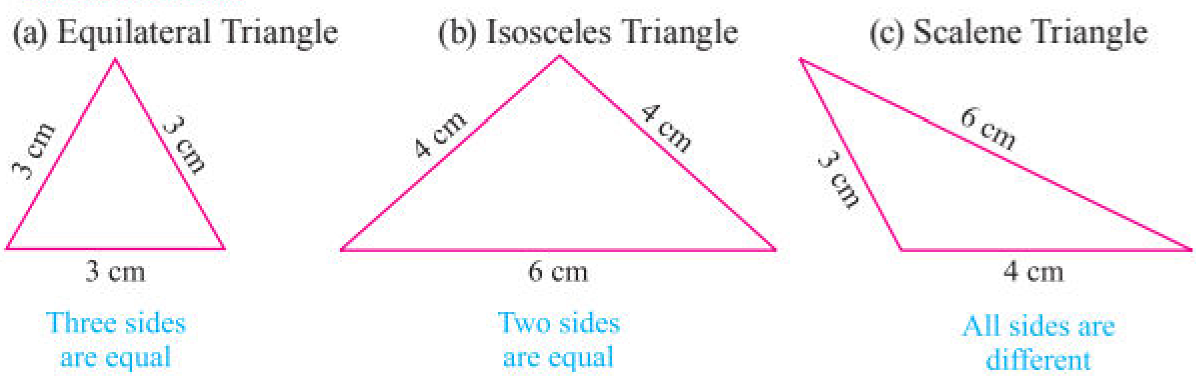
$$\therefore \text{Area of the shaded portion} = 168 \text{ cm}^2.$$

- The central angle of a circle is 360°.
- Perimeter of a semicircle = $(\pi + 2) \times r$ units.
- Area of a semicircle = $\frac{\pi r^2}{2}$ sq . units.
- The central angle of a semicircle is 180°.
- Perimeter of a quadrant = $\left(\frac{\pi}{2} + 2\right) \times r$ units.
- Area of a quadrant = $\frac{\pi r^2}{4}$ sq . units.
- The central angle of a quadrant is 90°.
- Perimeter of a combined figure is length of its boundary.
- A polygon is a closed plane figure formed by 'n' line segments.
- Regular polygons are polygons in which all the sides and angles are equal.
- Irregular polygons are combination of plane figures.

3.2.1. Kinds of Triangles

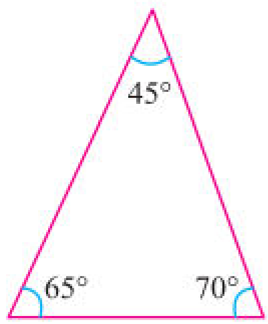
Triangles can be classified into two types based on sides and angles.

Based on sides:



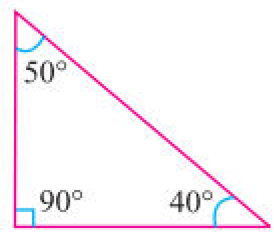
Based on angles:

(d) Acute Angled Triangle



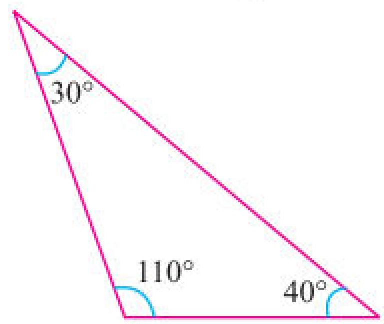
Three acute angles

(e) Right Angled Triangle



One right angle

(f) Obtuse Angled Triangle



One obtuse angle

Example 3.1

In ΔABC, $\angle A = 75°$, $\angle B = 65°$ find $\angle C$.

Solution

We know that in ΔABC,

$$\angle A + \angle B + \angle C = 180°$$

$$75° + 65° + \angle C = 180°$$

$$140° + \angle C = 180°$$

$$\angle C = 180° - 140°$$

$$\therefore \angle C = 40°.$$

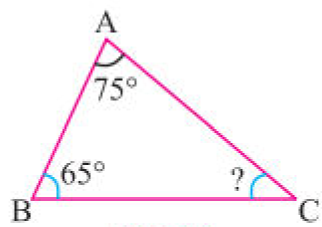


Fig. 3.9

Example 3.2

In ΔABC, given that $\angle A = 70^\circ$ and $AB = AC$. Find the other angles of ΔABC.

Solution

Let $\angle B = x^\circ$ and $\angle C = y^\circ$.

Given that ΔABC is an isosceles triangle.

$$\begin{aligned} AC &= AB \\ \angle B &= \angle C \quad [\text{Angles opposite to equal sides are equal}] \\ x^\circ &= y^\circ \end{aligned}$$

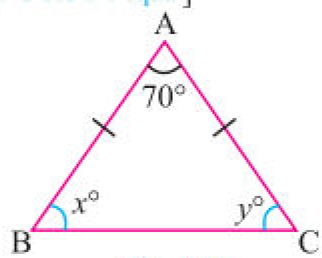


Fig. 3.10

$$\begin{aligned} \text{In } \Delta ABC, \angle A + \angle B + \angle C &= 180^\circ \\ 70^\circ + x^\circ + y^\circ &= 180^\circ \\ 70^\circ + x^\circ + x^\circ &= 180^\circ \quad [\because x^\circ = y^\circ] \\ 2x^\circ &= 180^\circ - 70^\circ \\ 2x^\circ &= 110^\circ \\ x^\circ &= \frac{110^\circ}{2} = 55^\circ. \end{aligned}$$

Hence $\angle B = 55^\circ$ and $\angle C = 55^\circ$.

Example 3.3

The measures of the angles of a triangle are in the ratio 5 : 4 : 3. Find the angles of the triangle.

Solution

Given that in a ΔABC, $\angle A : \angle B : \angle C = 5 : 4 : 3$.

Let the angles of the given triangle be $5x^\circ$, $4x^\circ$ and $3x^\circ$.

We know that the sum of the angles of a triangle is 180°.

$$5x^\circ + 4x^\circ + 3x^\circ = 180^\circ \Rightarrow 12x^\circ = 180^\circ$$

$$x^\circ = \frac{180^\circ}{12} = 15^\circ$$

So, the angles of the triangle are 75°, 60° and 45°.

Example 3.4

Find the angles of the triangle ABC, given in Fig.3.11.

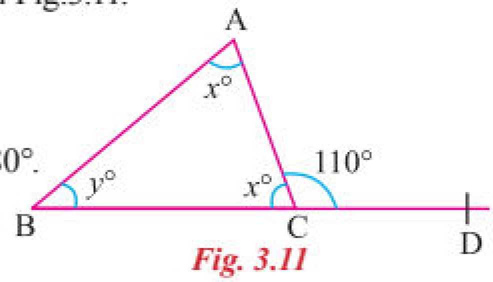


Fig. 3.11

Solution

BD is a straight line.

We know that angle in the line segment is 180°.

$$x^\circ + 110^\circ = 180^\circ$$
$$x^\circ = 180^\circ - 110^\circ$$
$$x^\circ = 70^\circ$$

We know that the exterior angle is equal to the sum of the two interior opposite angles.

$$x^\circ + y^\circ = 110^\circ$$
$$70^\circ + y^\circ = 110^\circ$$
$$y^\circ = 110^\circ - 70^\circ = 40^\circ$$

Hence, $x^\circ = 70^\circ$

and $y^\circ = 40^\circ$.

Example 1.3

(i) Subtract $5xy$ from $8xy$ (ii) Subtract $3c + 7d^2$ from $5c - d^2$

(iii) Subtract $2x^2 + 2y^2 - 6$ from $3x^2 - 7y^2 + 9$

Solution

(i) Subtract $5xy$ from $8xy$.

The first step is to place them as

$$\begin{array}{r} 8xy \\ -5xy \\ \hline 3xy \\ \hline \end{array}$$

(The two terms $8xy$, $-5xy$ are like terms)

$\therefore 8xy - 5xy = 3xy$

(ii) Subtract $3c + 7d^2$ from $5c - d^2$

Solution

$$\begin{array}{r} 5c - d^2 \\ -(3c + 7d^2) \\ \hline \\ \hline \end{array}$$

(or)

$$\begin{array}{r} 5c - d^2 \\ -3c - 7d^2 \\ \hline 2c - 8d^2 \\ \hline \end{array}$$

Often, we do this as →

$$\begin{array}{r} 5c - d^2 \\ 3c + 7d^2 \\ - \quad - \\ \hline 2c - 8d^2 \\ \hline \end{array}$$

Alternatively, this can also be done as :

$$\begin{aligned}(5c - d^2) - (3c + 7d^2) &= 5c - d^2 - 3c - 7d^2 \\ &= (5c - 3c) + (-d^2 - 7d^2) \\ &= 2c + (-8d^2) \\ &= 2c - 8d^2\end{aligned}$$

(iii) Subtract $2x^2 + 2y^2 - 6$ from $3x^2 - 7y^2 + 9$

Solution

$$\begin{array}{l} 3x^2 - 7y^2 + 9 \\ 2x^2 + 2y^2 - 6 \quad \text{[Change of the sign]} \\ - \quad - \quad + \\ \hline x^2 - 9y^2 + 15 \\ \hline \end{array}$$

Alternative Method

$(3x^2 - 7y^2 + 9) - (2x^2 + 2y^2 - 6)$

$= 3x^2 - 7y^2 + 9 - 2x^2 - 2y^2 + 6$

$= (3x^2 - 2x^2) + (-7y^2 - 2y^2) + (9 + 6)$

$= x^2 + (-9y^2) + 15$

$= x^2 - 9y^2 + 15$

(i) $x \times 5y = x \times 5 \times y = 5 \times x \times y = 5xy$

(ii) $2x \times 3y = 2 \times x \times 3 \times y = 2 \times 3 \times x \times y = 6xy$

(iii) $2x \times (-3y) = 2 \times (-3) \times x \times y = -6 \times x \times y = -6xy$

(iv) $2x \times 3x^2 = 2 \times x \times 3 \times x^2 = (2 \times 3) \times (x \times x^2) = 6x^3$

(v) $2x \times (-3xyz) = 2 \times (-3) \times (x \times xyz) = -6x^2yz.$

Note: 1. Product of monomials are also monomials.

2. Coefficient of the product = Coefficient of the first monomial $\times$ Coefficient of the second monomial.

3. Laws of exponents $a^m \times a^n = a^{m+n}$ is useful, in finding the product of the terms.

4. The products of a and b can be represented as: $a \times b$, ab, $a.b$, $a(b)$, $(a)\ b$, $(a)\ (b)$, (ab).

(vi) $(3x^2)(4x^3)$

$= (3 \times x \times x)(4 \times x \times x \times x)$

$= (3 \times 4)(x \times x \times x \times x \times x)$

$= 12x^5$

(Or)

$(3x^2)(4x^3) = (3 \times 4)(x^2 \times x^3) = 12(x^{2+3})$

$= 12x^5$ (using $a^m \times a^n = a^{m+n}$)

Some more useful examples are as follows:

(vii) $2x \times 3y \times 5z = (2x \times 3y) \times 5z$

$= (6xy) \times 5z$

$= 30\,xyz$

(or) $2x \times 3y \times 5z = (2 \times 3 \times 5) \times (x \times y \times z) = 30xyz$

(viii) $4ab \times 3a^2b^2 \times 2a^3b^3 = (4ab \times 3a^2b^2) \times 2a^3b^3$

$= (12a^3b^3) \times 2a^3\,b^3$

$= 24a^6b^6$

(or) $4ab \times 3a^2b^2 \times 2a^3b^3 = 4 \times 3 \times 2 \times (ab \times a^2b^2 \times a^3b^3)$

$= 24(a^{1+2+3} \times b^{1+2+3})$

$= 24a^6b^6$

1.3.2 Multiplying a Monomial by a Binomial

Let us learn to multiply a monomial by a binomial through the following examples.

Example 1.4

Simplify: $(2x) \times (3x + 5)$

Solution We can write this as:

Step 1

$(2x) \times (3x + 5) = (2x \times 3x) + (2x \times 5)$ [Using the distributive law]

Step 2

$= 6x^2 + 10x$

Example 1.5

Simplify: $(-2x) \times (4 - 5y)$

Solution

Step 1

$(-2x) \times (4 - 5y) = [(-2x) \times 4] + [(-2x) \times (-5y)]$

Step 2

$= (-8x) + (10xy)$ [Using the distributive law]

$= -8x + 10xy$

Note: (i) The product of a monomial by a binomial is a binomial.

(ii) We use the commutative and distributive laws to solve multiplication sums. $a \times b = b \times a$ **(Commutative Law)**

$a(b + c) = ab + ac$ and $a(b - c) = ab - ac$ **(Distributive laws)**

1.3.3. Multiplying a Monomial by a Polynomial

A Polynomial with more than two terms is multiplied by a monomial as follows:

Example 1.6

Simplify: (i) $3(5y^2 - 3y + 2)$

(ii) $2x^2 \times (3x^2 - 5x + 8)$

Solution

(i) $3(5y^2 - 3y + 2) = (3 \times 5y^2) + (3 \times -3y) + (3 \times 2)$

$= 15y^2 - 9y + 6$

[or]

$$\begin{array}{r} 5y^2 - 3y + 2 \\ \times \quad\quad\quad 3 \\ \hline 15y^2 - 9y + 6 \\ \hline \end{array}$$

(ii) $2x^2 \times (3x^2 - 5x + 8)$

$= (2x^2 \times 3x^2) + (2x^2 \times (-5x)) + (2x^2 \times 8)$

$= 6x^4 - 10x^3 + 16x^2$

[or]

$$\begin{array}{r} 3x^2 - 5x + 8 \\ \times \quad\quad 2x^2 \\ \hline 6x^4 - 10x^3 + 16x^2 \\ \hline \end{array}$$

1.3.4 Multiplying a Binomial by a Binomial

We shall now proceed to multiply a binomial by another binomial, using the distributive and commutative laws. Let us consider the following example.

Example 1.7

Simplify : $(2a + 3b)(5a + 4b)$

Solution

Every term in one binomial multiplies every term in the other binomial.

$$
\begin{aligned}
(2a + 3b)\ (5a + 4b) &= (2a \times 5a) + (2a \times 4b) + (3b \times 5a) + (3b \times 4b) \\
&= 10a^2 + 8ab + 15ba + 12b^2 \\
&= 10a^2 + 8ab + 15ab + 12b^2 \qquad [\because ab = ba] \\
&= 10a^2 + 23ab + 12b^2 \\
&\quad \text{[Adding like terms } 8ab \text{ and } 15ab] \\
(2a + 3b)(5a + 4b) &= 10a^2 + 23ab + 12b^2
\end{aligned}
$$

Note : In the above example, while multiplying two binomials we get only 3 terms instead of $2 \times 2 = 4$ terms. Because we have combined the like terms $8ab$ and $15ab$.

1.3.5 Multiplying a Binomial by a Trinomial

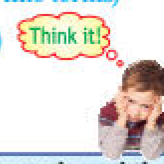
In this multiplication, we have to multiply each of the three terms of the trinomial by each of the two terms in the binomial.

Example 1.8

Simplify: $(x+3)(x^2-5x+7)$

Solution

$$\begin{aligned}(x+3)\ (x^2-5x+7) &= x\ (x^2-5x+7) + 3\ (x^2-5x+7) && \text{(Using the distributive law)}\\ &= x^3-5x^2+7x+3x^2-15x+21\\ &= x^3-5x^2+3x^2+7x-15x+21 && \text{(Grouping the like terms)}\\ &= x^3-2x^2-8x+21 && \text{(Combining the like terms)}\end{aligned}$$



Alternative Method :

$$\begin{array}{lcr} & & (x+3)\\ & & \times\ (x^2-5x+7)\\ x(x^2-5x+7) & : & x^3-5x^2+\ \ 7x\qquad\qquad\\ 3(x^2-5x+7) & : & 3x^2-15x+21\\ \hline & & = x^3-2x^2-8x+21\\ \hline \end{array}$$

In this example, while multipltying, instead of expecting $2 \times 3 = 6$ terms, we are getting only 4 terms in the product. **Could you find out the reason?**

1.4.1 Algebraic Identities

We proceed now to study the three important Algebraic Identities which are very useful in solving many problems. We obtain these Identities by multiplying a binomial by another binomial.

Identity 1

Let us consider $(a+b)^2$.

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2+ab+ba+b^2\\ &= a^2+ab+ab+b^2 && [\because ab = ba]\\ &= a^2+2ab+b^2\end{aligned}$$

Thus, $\boxed{(a+b)^2 = a^2+2ab+b^2}$

1.4.2 Applying the Identities

Example 1.9

Expand (i) $(x+5)^2$ (ii) $(x+2y)^2$ (iii) $(2x+3y)^2$ (iv) 105^2.

Solution

(i)
$$\begin{aligned}(x+5)^2 &= x^2+2(x)(5)+5^2\\ &= x^2+10x+25\end{aligned}$$

Using the identity:
$(a+b)^2 = a^2+2ab+b^2$
Here, $a = x$, $b = 5$.

Aliter:
$$\begin{aligned}(x+5)^2 &= (x+5)(x+5)\\ &= x(x+5)+5(x+5)\\ &= x^2+5x+5x+25\\ &= x^2+10x+25\end{aligned}$$

(ii)
$$\begin{aligned}(x+2y)^2 &= x^2+2(x)(2y)+(2y)^2\\ &= x^2+4xy+4y^2\end{aligned}$$

Using the identity:
$(a+b)^2 = a^2+2ab+b^2$
Here, $a = x$, $b = 2y$.

Aliter:
$$\begin{aligned}(x+2y)^2 &= (x+2y)(x+2y)\\ &= x(x+2y)+2y(x+2y)\\ &= x^2+2xy+2yx+4y^2 && [\because xy = yx]\\ &= x^2+4xy+4y^2\end{aligned}$$

(iii)
$$\begin{aligned}(2x+3y)^2 &= (2x)^2+2(2x)(3y)+(3y)^2\\ &= 4x^2+12xy+9y^2\end{aligned}$$

Using the identity:
$(a+b)^2 = a^2+2ab+b^2$
Here, $a = 2x$, $b = 3y$.

Aliter:
$$\begin{aligned}(2x+3y)^2 &= (2x+3y)(2x+3y)\\ &= 2x(2x+3y)+3y(2x+3y)\\ &= (2x)(2x)+(2x)(3y)+(3y)(2x)+(3y)(3y)\\ &= 4x^2+6xy+6yx+9y^2 && [\because xy = yx]\\ (2x+3y)^2 &= 4x^2+12xy+9y^2\end{aligned}$$

(iv)
$$\begin{aligned}105^2 &= (100+5)^2\\ &= 100^2+2(100)(5)+5^2\\ &= (100\times 100)+1000+25\\ &= 10000+1000+25\\ 105^2 &= 11025\end{aligned}$$

Using the identity:
$(a+b)^2 = a^2+2ab+b^2$
Here, $a = 100$, $b = 5$.

Example 1.10

Find the values of (i) $(x-y)^2$ (ii) $(3p-2q)^2$ (iii) 97^2 (iv) $(4.9)^2$

Solution

(i)

$$(x-y)^2 = x^2 - 2(x)(y) + y^2$$
$$= x^2 - 2xy + y^2$$

Using the identity:
$(a-b)^2 = a^2 - 2ab + b^2$
Here, $a = x$, $b = y$.

(ii)

$$(3p-2q)^2 = (3p)^2 - 2(3p)(2q) + (2q)^2$$
$$= 9p^2 - 12pq + 4q^2$$

Using the identity:
$(a-b)^2 = a^2 - 2ab + b^2$
Here, $a = 3p$, $b = 2q$.

(iii)

$$97^2 = (100-3)^2$$
$$= (100)^2 - 2(100)(3) + 3^2$$
$$= 10000 - 600 + 9$$
$$= 9400 + 9$$
$$= 9409$$

Using the identity:
$(a-b)^2 = a^2 - 2ab + b^2$
Here, $a = 100$, $b = 3$.

(iv)

$$(4.9)^2 = (5.0-0.1)^2$$
$$= (5.0)^2 - 2(5.0)(0.1) + (0.1)^2$$
$$= 25.00 - 1.00 + 0.01$$
$$= 24.01$$

Using the identity:
$(a-b)^2 = a^2 - 2ab + b^2$
Here, $a = 5.0$, $b = 0.1$.

Example 1.11

Evaluate the following using the identity $(a+b)(a-b)=a^2-b^2$

(i) $(x+3)(x-3)$ (ii) $(5a+3b)(5a-3b)$ (iii) 52×48 (iv) 997^2-3^2.

Solution

(i)
$$(x+3)(x-3) = x^2-3^2 = x^2-9$$

Using the identity: $(a+b)(a-b)=a^2-b^2$. Here, $a=x$, $b=3$.

(ii)
$$(5a+3b)(5a-3b) = (5a)^2-(3b)^2 = 25a^2-9b^2$$

Using the identity: $(a+b)(a-b)=a^2-b^2$. Here, $a=5a$, $b=3b$.

(iii)
$$52\times 48 = (50+2)(50-2) = 50^2-2^2 = 2500-4 = 2496$$

Using the identity: $(a+b)(a-b)=a^2-b^2$. Here, $a=50$, $b=2$.

(iv)
$$997^2-3^2 = (997+3)(997-3) = (1000)(994) = 994000$$

Using the identity: $a^2-b^2=(a+b)(a-b)$. Here, $a=997$, $b=3$.

1.4.3 Deducing some useful Identities

Let us consider,

(i)
$$\begin{aligned}(a+b)^2+(a-b)^2 &= (a^2+2ab+b^2)+(a^2-2ab+b^2)\\ &= a^2+\cancel{2ab}+b^2+a^2-\cancel{2ab}+b^2\\ &= 2a^2+2b^2\\ (a+b)^2+(a-b)^2 &= 2(a^2+b^2)\end{aligned}$$

$$\boxed{\frac{1}{2}[(a+b)^2+(a-b)^2] = a^2+b^2}$$

(ii)
$$\begin{aligned}(a+b)^2-(a-b)^2 &= (a^2+2ab+b^2)-(a^2-2ab+b^2)\\ &= \cancel{a^2}+2ab+\cancel{b^2}-\cancel{a^2}+2ab-\cancel{b^2}\\ (a+b)^2-(a-b)^2 &= 4ab\end{aligned}$$

$$\boxed{\frac{1}{4}[(a+b)^2-(a-b)^2] = ab}$$

(iii) $(a+b)^2 - 2ab = a^2 + b^2 + \cancel{2ab} - \cancel{2ab}$

$= a^2 + b^2$

$(a+b)^2 - 2ab = a^2 + b^2$

(iv) $(a+b)^2 - 4ab = a^2 + 2ab + b^2 - 4ab$

$= a^2 - 2ab + b^2$

$= (a-b)^2$

$(a+b)^2 - 4ab = (a-b)^2$

(v) $(a-b)^2 + 2ab = a^2 - \cancel{2ab} + b^2 + \cancel{2ab}$

$= a^2 + b^2$

$(a-b)^2 + 2ab = a^2 + b^2$

(vi) $(a-b)^2 + 4ab = a^2 - 2ab + b^2 + 4ab$

$= a^2 + 2ab + b^2$

$= (a+b)^2$

$(a-b)^2 + 4ab = (a+b)^2$

Deduced Identities

- $\frac{1}{2}[(a+b)^2 + (a-b)^2] = a^2 + b^2$
- $\frac{1}{4}[(a+b)^2 - (a-b)^2] = ab$
- $(a+b)^2 - 2ab = a^2 + b^2$
- $(a+b)^2 - 4ab = (a-b)^2$
- $(a-b)^2 + 2ab = a^2 + b^2$
- $(a-b)^2 + 4ab = (a+b)^2$

If the values of $a+b$ and $a-b$ are 7 and 4 respectively, find the values of a^2+b^2 and ab.

Solution

(i)
$$
\begin{aligned}
a^2+b^2 &= \frac{1}{2}[(a+b)^2+(a-b)^2] \\
&= \frac{1}{2}[7^2+4^2] \quad \text{[Substituting the values of } a+b=7,\ a-b=4] \\
&= \frac{1}{2}(49+16) \\
&= \frac{1}{2}(65) \\
&= \frac{65}{2} \\
a^2+b^2 &= \frac{65}{2}
\end{aligned}
$$

(ii)
$$
\begin{aligned}
ab &= \frac{1}{4}[(a+b)^2-(a-b)^2] \\
&= \frac{1}{4}(7^2-4^2) \quad \text{[Substituting the values of } a+b=7,\ a-b=4] \\
&= \frac{1}{4}(49-16) \\
&= \frac{1}{4}(33) \\
ab &= \frac{33}{4}
\end{aligned}
$$

Example 1.14

If $(a+b)=10$ and $ab=20$, find a^2+b^2 and $(a-b)^2$.

Solution

(i)
$$
\begin{aligned}
a^2+b^2 &= (a+b)^2-2ab \quad \text{[Substituting } a+b=10,\ ab=20] \\
a^2+b^2 &= (10)^2-2(20) \\
&= 100-40 = 60 \\
a^2+b^2 &= 60
\end{aligned}
$$

(ii)
$$
\begin{aligned}
(a-b)^2 &= (a+b)^2-4ab \quad \text{[Substituting } a+b=10,\ ab=20] \\
&= (10)^2-4(20) \\
&= 100-80 \\
(a-b)^2 &= 20
\end{aligned}
$$

Example 1.15

If $(x+l)(x+m) = x^2 + 4x + 2$ find $l^2 + m^2$ and $(l-m)^2$

Solution

By product formula, we know

$$(x+l)(x+m) = x^2 + (l+m)x + lm$$

So, by comparing RHS with $x^2 + 4x + 2$, we have,

$$l + m = 4 \text{ and } lm = 2$$

Now,
$$\begin{aligned} l^2 + m^2 &= (l+m)^2 - 2lm \\ &= 4^2 - 2(2) = 16 - 4 \\ l^2 + m^2 &= 12 \\ (l-m)^2 &= (l+m)^2 - 4lm \\ &= 4^2 - 4(2) = 16 - 8 \\ (l-m)^2 &= 8 \end{aligned}$$

Example 1.21

Factorize: $x^2 + 6x + 8$

Solution

Comparing $x^2 + 6x + 8$ with $x^2 + (a+b)x + ab = (x+a)(x+b)$,

we get $ab = 8$ and $a + b = 6$.

$$\begin{aligned} \therefore\ x^2 + 6x + 8 &= x^2 + (2+4)x + (2 \times 4) \\ &= (x+2)(x+4) \end{aligned}$$

The factors of $x^2 + 6x + 8$ are $(x+2)$ and $(x+4)$.

Factors of 8	Sum of factors
1, 8	9
2, 4	6

Hence the correct factors are 2, 4

Solve: $(5x^2 + 10x) \div (x + 2)$.

Solution

$$(5x^2 + 10x) \div (x + 2) = \frac{5x^2 + 10x}{x + 2}$$

Let us factorize the numerator $(5x^2 + 10x)$.

$$5x^2 + 10x = (5 \times x \times x) + (5 \times 2 \times x)$$

$$= 5x(x + 2) \quad \text{[Taking out the common factor } 5x\text{]}$$

Now, $$(5x^2 + 10x) \div (x + 2) = \frac{5x^2 + 10x}{x + 2}$$

$$= \frac{5x\cancel{(x+2)}}{\cancel{(x+2)}} = 5x. \quad \text{[By cancelling } (x+2)\text{]}$$

Example 1.26

Solve: $2x + 5 = 23 - x$

Solution

$$2x + 5 = 23 - x$$

$$2x + 5 - 5 = 23 - x - 5$$

[Adding – 5 both sides]

$$2x = 18 - x$$

$$2x + x = 18 - x + x$$

$$3x = 18$$

$$\frac{3x}{3} = \frac{18}{3} \quad \text{[Dividing both the sides by 3]}$$

$$x = 6$$

Alternative Method

$$2x + 5 = 23 - x$$

$$2x + x = 23 - 5 \quad \text{[By transposition]}$$

$$3x = 18$$

$$x = \frac{18}{3} \quad \text{[Dividing both sides by 3]}$$

$$x = 6$$

Verification: LHS = $2x + 5 = 2(6) + 5 = 17$,

RHS = $23 - x = 23 - 6 = 17$.

Example 1.27

Solve : $\frac{9}{2}m + m = 22$

Solution

$$\frac{9}{2}m + m = 22$$

$$\frac{9m+2m}{2} = 22 \quad \text{[Taking LCM on LHS]}$$

$$\frac{11m}{2} = 22$$

$$m = \frac{22 \times 2}{11} \quad \text{[By cross multiplication]}$$

$$m = 4$$

Verification:

$$\text{LHS} = \frac{9}{2}m + m = \frac{9}{2}(4) + 4$$

$$= 18 + 4 = 22 = \text{RHS}$$

Example 1.28

Solve: $\frac{2}{x} - \frac{5}{3x} = \frac{1}{9}$

Solution

$$\frac{2}{x} - \frac{5}{3x} = \frac{1}{9}$$

$$\frac{6-5}{3x} = \frac{1}{9} \quad \text{[Taking LCM on LHS]}$$

$$\frac{1}{3x} = \frac{1}{9}$$

$$3x = 9;\ x = \frac{9}{3};\ x = 3.$$

Verification:

$$\text{LHS} = \frac{2}{x} - \frac{5}{3x}$$

$$= \frac{2}{3} - \frac{5}{3(3)} = \frac{2}{3} - \frac{5}{9}$$

$$= \frac{6-5}{9} = \frac{1}{9} = \text{RHS}$$

Example 1.29

Find the two consecutive positive odd integers whose sum is 32.

Solution

Let the two consecutive positive odd integers be x and $(x+2)$.

Then, their sum is 32.

$$\therefore (x) + (x+2) = 32$$

$$2x + 2 = 32$$

$$2x = 32 - 2$$

$$2x = 30$$

$$x = \frac{30}{2} = 15$$

Since $x = 15$, then the other integer, $x + 2 = 15 + 2 = 17$

Verification:

$15 + 17 = 32$

$\therefore$ The two required consecutive positive odd integers are 15 and 17.

Example 1.30

One third of one half of one fifth of a number is 15. Find the number.

Solution

Let the required number be x.

$$\text{Then, } \frac{1}{3} \text{ of } \frac{1}{2} \text{ of } \frac{1}{5} \text{ of } x = 15.$$

$$\text{i.e. } \frac{1}{3}\times\frac{1}{2}\times\frac{1}{5}\times x = 15$$

$$x = 15\times3\times2\times5$$

$$x = 45 \times 10 = 450$$

Hence the required number is 450.

Verification:

$$\text{LHS} = \frac{1}{3}\times\frac{1}{2}\times\frac{1}{5}\times x$$

$$= \frac{1}{3}\times\frac{1}{2}\times\frac{1}{5}\times 450$$

$$= 15 = \text{RHS}$$

Example 1.31

A rational number is such that when we multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product we get $\frac{-7}{12}$. What is the number?

Solution

Let the rational number be x.

When we multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product we get $\frac{-7}{12}$.

$$\text{i.e., } x\times\frac{5}{2}+\frac{2}{3} = \frac{-7}{12}$$

$$\frac{5x}{2} = \frac{-7}{12} - \frac{2}{3}$$

$$= \frac{-7-8}{12}$$

$$= \frac{-15}{12}$$

$$x = \frac{-15}{12}\times\frac{2}{5}$$

$$= \frac{-1}{2}.$$

Hence the required number is $\frac{-1}{2}$.

Verification:

$$\text{LHS} = \frac{-1}{2}\times\frac{5}{2}+\frac{2}{3} = \frac{-5}{4}+\frac{2}{3}$$

$$= \frac{-15+8}{12} = \frac{-7}{12} = \text{RHS.}$$

Example 1.32

Arun is now half as old as his father. Twelve years ago the father's age was three times as old as Arun. Find their present ages.

Solution

Let Arun's age be x years now.

Then his father's age $= 2x$ years

12 years ago, Arun's age was $(x-12)$ years and

his faher's age was $(2x-12)$ years.

Given that,

$$\begin{aligned}(2x-12) &= 3(x-12)\\ 2x-12 &= 3x-36\\ 36-12 &= 3x-2x\\ x &= 24\end{aligned}$$

Therefore, Arun's present age $= 24$ years.

His father's present age $= 2(24) = 48$ years.

Verification:

Arun's age	Father's age
Now : 24	48
12 years ago $24-12 = 12$	$48-12=36$ $36 = 3$ (Arun's age) $= 3(12) = 36$

Example 1.33

By selling a car for ₹ 1,40,000, a man suffered a loss of 20%. What was the cost price of the car?

Solution

Let the cost price of the car be x.

Loss of 20% $= \frac{20}{100}$ of $x = \frac{1}{5}\times x = \frac{x}{5}$

We know that,

$$\begin{aligned}\text{Cost price} - \text{Loss} &= \text{Selling price}\\ x-\frac{x}{5} &= 140000\\ \frac{5x-x}{5} &= 140000\\ \frac{4x}{5} &= 140000\\ x &= 140000\times\frac{5}{4}\\ x &= 175000\end{aligned}$$

Hence the cost price of the car is ₹ 1,75,000.

Verification:

Loss $=$ 20% of 175000

$= \frac{20}{100}\times 175000$

$=$ ₹ 35,000

S.P $=$ C.P - Loss

$= 175000 - 35000$

$= 140000$

RESULTS ON PROFIT, LOSS AND SIMPLE INTEREST

(i) Profit or Gain = Selling price – Cost price

(ii) Loss = Cost price – Selling price

(iii) Profit % = $\frac{\text{Profit}}{\text{C.P.}} \times 100.$

(iv) Loss % = $\frac{\text{Loss}}{\text{C.P.}} \times 100$

(v) Simple interest (I) = $\frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100} = \frac{Pnr}{100}$

(vi) Amount = Principal + Interest

1.3.1. Application of Percentage

We have already learnt percentages in the previous classes. We present these ideas as follows:

(i) Two percent = $2\% = \frac{2}{100}$

(ii) 8% of 600 kg = $\frac{8}{100} \times 600 = 48$ kg

(ii) 125% = $\frac{125}{100} = \frac{5}{4} = 1\frac{1}{4}$

Do you know?

$= \frac{1}{2} = 50\ \%$

$= \frac{1}{4} = 25\ \%$

$= \frac{3}{4} = 75\ \%$

Now, we learn to apply percentages in some problems.

Example 1.1

What percent is 15 paise of 2 rupees 70 paise?

Solution

2 rupees 70 paise = (2 × 100 paise + 70 paise)

= 200 paise + 70 paise

= 270 paise

Required percentage = $\frac{15}{270} \times 100 = \frac{50}{9} = 5\frac{5}{9}\%.$

Example 1.2

Find the total amount if 12% of it is ₹ 1080.

Solution

Let the total amount be x .

Given : 12% of the total amount = ₹ 1080

$$\frac{12}{100} \times x = 1080$$

$$x = \frac{1080 \times 100}{12} = ₹\ 9000$$

$\therefore$ The total amount = ₹ 9000.

Example 1.3

72% of 25 students are good in Mathematics. How many are not good in Mathematics?

Solution

Percentage of students good in Mathematics = 72%

Number of students good in Mathematics = 72% of 25 students

$$= \frac{72}{100} \times 25 = 18 \text{ students}$$

Number of students not good in Mathematics = $25 - 18 = 7$.

Example 1.4

Find the number which is 15% less than 240.

Solution

$$15\% \text{ of } 240 = \frac{15}{100} \times 240 = 36$$

$\therefore$ The required number = $240 - 36 = 204$.

Example 1.5

The price of a house is decreased from Rupees Fifteen lakhs to Rupees Twelve lakhs. Find the percentage of decrease.

Solution

Original price = ₹ 15,00,000

Change in price = ₹ 12,00,000

Decrease in price = 15,00,000 – 12,00,000 = 3,00,000

$$\therefore \text{Percentage of decrease} = \frac{300000}{1500000} \times 100 = 20\%$$

Remember

$$\text{Percentage of increase} = \frac{\text{Increase in amount}}{\text{Original amount}} \times 100$$

$$\text{Percentage of decrease} = \frac{\text{Decrease in amount}}{\text{Original amount}} \times 100$$

(i) Illustration of the formula for S.P.

Consider the following situation:

Rajesh buys a pen for ₹ 80 and sells it to his friend. If he wants to make a profit of 5%, can you say the price for which he would have sold?

Rajesh bought the pen for ₹ 80 which is the Cost Price (C.P.). When he sold, he makes a profit of 5% which is calculated on the C.P.

$$\therefore \text{Profit} = 5\% \text{ of C.P.} = \frac{5}{100} \times 80 = ₹\ 4$$

Since there is a profit, S.P. > C.P.

$$\textbf{S.P.} = \textbf{C.P.} + \textbf{Profit}$$

$$= 80 + 4 = ₹\ 84.$$

$\therefore$ The price for which Rajesh would have sold = ₹ 84.

The same problem can be done using the formula.

$$\text{Selling price (S.P.)} = \frac{\mathbf{(100 + Profit\%)}}{\mathbf{100}} \times \textbf{C.P.}$$

$$= \frac{(100+5)}{100} \times 80 = \frac{105}{100} \times 80 = ₹\ 84.$$

(ii) Illustration of the formula for C.P.

Consider the following situation:

Suppose a shopkeeper sells a wrist watch for ₹ 540 making a profit of 5 %, then what would have been the cost of the watch?

The shopkeeper had sold the watch at a profit of 5 % on the C.P. Since C.P. is not known, let us take it as ₹ 100.

Profit of 5% is made on the C.P.

$$\therefore \text{Profit} = 5\% \text{ of C.P.}$$
$$= \frac{5}{100} \times 100$$
$$= ₹\ 5.$$

We know,

$$\textbf{S.P.} = \textbf{C.P.} + \textbf{Profit}$$
$$= 100 + 5$$
$$= ₹\ 105.$$

Here , if S.P. is ₹ 105, then C.P. is ₹ 100.

When S.P. of the watch is ₹ 540, C.P. $= \frac{540 \times 100}{105} =$ ₹ 514.29

$\therefore$ The watch would have cost ₹ 514.29 for the shopkeeper.

The above problem can also be solved by using the formula method.

$$\textbf{C.P.} = \left(\frac{\mathbf{100}}{\mathbf{100 + profit\ \%}}\right) \times \textbf{S.P.}$$
$$= \frac{100}{100+5} \times 540$$
$$= \frac{100}{105} \times 540$$
$$= ₹\ 514.29.$$

We now summarize the formulae to calculate S.P. and C.P. as follows:

1. When there is a profit (i) C.P. = $\left(\dfrac{100}{100 + \text{profit\%}}\right) \times$ S.P.	1. When there is a loss (ii) C.P. = $\left(\dfrac{100}{100 - \text{loss\%}}\right) \times$ S.P.
2. When there is a profit (i) S.P. = $\left(\dfrac{100 + \text{profit \%}}{100}\right) \times$ C.P.	2. When there is a loss, (ii) S.P. = $\left(\dfrac{100 - \text{loss\%}}{100}\right) \times$ C.P.

Example 1.6

Hameed buys a colour T.V set for ₹ 15,200 and sells it at a loss of 20%. What is the selling price of the T.V set?

Solution

Raghul used this method:

Loss is 20% of the C.P.

$= \dfrac{20}{100} \times 15200$

$=$ ₹ 3040

S.P. = C.P. – Loss

= 15,200 – 3,040

= ₹ 12,160

OR

Roshan used the formula method:

C.P. = ₹ 15, 200

Loss = 20%

S.P. $= \dfrac{100 - \text{Loss\%}}{100} \times$ C.P.

$= \dfrac{100 - 20}{100} \times 15200$

$= \dfrac{80}{100} \times 15200$

= ₹ 12,160

Both Raghul and Roshan came out with the same answer that the selling price of the T.V. set is ₹ 12,160.

Example 1.7

A scooty is sold for ₹ 13,600 and fetches a loss of 15%. Find the cost price of the scooty.

Devi used this method:

Loss of 15% means,

if C.P. is ₹ 100, Loss = ₹ 15

Therefore, S.P. would be

$(100 - 15) = ₹\ 85$

If S.P. is ₹ 85, C.P. is ₹ 100

When S.P. is ₹13,600, then

$$\text{C.P.} = \frac{100 \times 13600}{85} = ₹\ 16{,}000$$

OR

Revathi used the formula method:

$$\text{Loss} = 15\%.$$

$$\text{S.P.} = ₹\ 13{,}600$$

$$\text{C.P.} = \frac{100}{100 - \text{Loss}\%} \times \text{S.P.}$$

$$= \frac{100}{100 - 15} \times 13600$$

$$= \frac{100}{85} \times 13600$$

$$= ₹\ 16{,}000$$

Hence the cost price of the scooty is ₹ 16,000.

Example 1.8

The cost price of 11 pens is equal to the selling price of 10 pens. Find the loss or gain percent.

Solution

Let S.P. of each pen be x.

$$\text{S.P. of 10 pens} = ₹\ 10x$$

$$\text{S.P. of 11 pens} = ₹\ 11x$$

Given : C.P. of 11 pens = S.P. of 10 pens = ₹ $10x$

Here, S.P. > C.P.

$$\therefore \text{Profit} = \text{S.P.} - \text{C.P.}$$

$$= 11x - 10x \quad = x$$

$$\text{Profit }\% = \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{x}{10x} \times 100 = 10\%.$$

Example 1.9

A man sells two wrist watches at ₹ 594 each. On one he gains 10% and on the other he loses 10%. Find his gain or loss percent on the whole.

Solution

Given : S.P. of the first wrist watch $= ₹\ 594$, Gain% = 10%

$$\therefore \text{ C.P. of the first wrist watch} = \frac{100}{100 + \text{profit}\%} \times \text{S.P.}$$

$$= \frac{100}{(100+10)} \times 594$$

$$= \frac{100}{110} \times 594 = ₹\ 540.$$

Similarly, C.P. of the second watch on which he loses 10% is

$$= \frac{100}{(100 - \text{Loss}\%)} \times \text{S.P.}$$

$$= \frac{100}{(100-10)} \times 594 = \frac{100}{90} \times 594 = ₹\ 660.$$

To say whether there was an overall Profit or Loss, we need to find the combined C.P. and S.P.

$$\text{Total C.P. of the two watches} = 540 + 660 = ₹\ 1{,}200.$$

$$\text{Total S.P. of the two watches} = 594 + 594 = ₹\ 1{,}188.$$

$$\text{Net Loss} = 1{,}200 - 1{,}188 = ₹\ 12.$$

$$\text{Loss}\% = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

$$= \frac{12}{1200} \times 100 = 1\%.$$

Example 1.10

Raju bought a motorcycle for ₹ 36,000 and then bought some extra fittings to make it perfect and good looking. He sold the bike at a profit of 10% and he got ₹ 44,000. How much did he spend to buy the extra fittings made for the motorcycle?

Solution

Let the C.P. be ₹ 100.

$$\text{Profit} = 10\%, \text{ S.P.} = ₹\ 110$$

If S.P. is ₹ 110, then C.P. is ₹ 100.

When S.P. is ₹ 44,000

$$\text{C.P.} = \frac{44000 \times 100}{110} = ₹\ 40,000$$

∴ Amount spent on extra fittings = 40,000 – 36,000 = ₹ 4,000.

1.3.3. Application of Overhead Expenses

Maya went with her father to purchase an Air cooler. They bought it for ₹ 18,000. The shop wherein they bought was not closer to their residence. So they had to arrange a vehicle to take the air cooler to their residence. They paid conveyance charges of ₹ 500. Hence the C.P. of the air cooler is not only ₹ 18,000 but it also includes the **Conveyance Charges** (Transportation charges) ₹ 500 which is called as **Overhead Expenses .**

Now,

$$\begin{aligned}\text{C.P. of the air cooler} &= \text{Real cost} + \text{Conveyance charges}\\ &= 18,000 + 500 = ₹\ 18,500\end{aligned}$$

Consider another situation, where Kishore's father buys an old Maruti car from a Chennai dealer for ₹ 2,75,000 and spends ₹ 25,000 for painting the car. And then he transports the car to his native village for which he spends again ₹ 2,000. Can you answer the following questions:

(i) What is the the overall cost price of the car?

(ii) What is the real cost price of the car?

(iii) What are the overhead expenses referred here?

In the above example the painting charges and the transportation charges are the overhead expenses.

∴ Cost price of the car = Real cost price + Overhead expenses

= 2,75,000 + (25,000 + 2,000)

= 2,75,000 + 27,000 = ₹ 3,02,000.

Sometimes when an article is bought or sold, some additional expenses occur while buying or before selling it. These expenses have to be included in the cost price. These expenses are referred to as **Overhead Expenses**. These may include expenses like amount spent on repairs, labour charges, transportation, etc.,

Discount = Marked Price – Selling Price

Selling Price = Marked Price – Discount

Marked Price = Selling Price + Discount

Example 1.11

A bicycle marked at ₹ 1,500 is sold for ₹ 1,350. What is the percentage of discount?

Solution

Given : Marked Price = ₹ 1500, Selling Price = ₹ 1350

Amount of discount = Marked Price – Selling Price

= 1500 – 1350

= ₹ 150

Discount for ₹ 1500 = ₹ 150

Discount for ₹ 100 = $\frac{150}{1500} \times 100$

Percentage of discount = 10%.

Since Discount is on Marked Price, we will have calculate discount on M.P.

Example 1.12

The list price of a frock is ₹ 220. A discount of 20% on sales is announced. What is the amount of discount on it and its selling price?

Solution

Given : List (Marked) Price of the frock = ₹ 220, Rate of discount = 20%

Amount of discount = $\frac{20}{100} \times 220$

= ₹ 44

∴ Selling Price of the frock = Marked Price – Discount

= 220 – 44

= ₹ 176.

Example 1.13

An almirah is sold at ₹ 5,225 after allowing a discount of 5%. Find its marked price.

Solution

Krishna used this method:

The discount is given in percentage.

Hence, the M.P. is taken as ₹ 100.

Rate of discount = 5%

Amount of discount $= \frac{5}{100} \times 100$

= ₹ 5.

Selling Price = M.P. – Discount

= 100 – 5 = ₹ 95

If S.P. is ₹ 95, then M.P. is ₹ 100.

When S.P. is ₹ 5225,

M.P. $= \frac{100}{95} \times 5225$

∴ M.P. of the almirah = ₹ 5,500.

[OR]

Vignesh used the formula method:

S.P. = Rs 5225

Discount = 5%

M.P. = ?

$$\text{M.P.} = \left(\frac{100}{100 - \text{Discount\%}}\right) \times \text{S.P.}$$

$$= \left(\frac{100}{100 - 5}\right) \times 5225$$

$$= \frac{100}{95} \times 5225$$

= ₹ 5,500.

Example 1.14

A shopkeeper allows a discount of 10% to his customers and still gains 20%. Find the marked price of an article which costs ₹ 450 to the shopkeeper.

Solution

Vanitha used this method:

Let M.P. be ₹ 100.

Discount = 10% of M. P.

$= \frac{10}{100}$ of M.P. $= \frac{10}{100} \times 100$

= ₹ 10

S.P. = M.P. – Discount

= 100 – 10 = ₹ 90

Gain = 20% of C.P.

$= \frac{20}{100} \times 450 =$ ₹ 90

S.P. = C.P. + Gain

= 450 + 90 = ₹ 540.

If S.P. is ₹ 90, then M.P. is ₹ 100.

When S.P. is ₹ 540,

M.P. $= \frac{540 \times 100}{90} =$ ₹ 600

$\therefore$ The M.P. of an article = ₹ 600

[OR]

Vimal used the formula method:

Discount = 10%, Gain = 20%,

C.P. = ₹ 450, M.P. = ?

$$\text{M.P.} = \frac{100 + \text{Gain\%}}{100 - \text{Discount\%}} \times \text{C.P.}$$

$$= \frac{(100+20)}{(100-10)} \times 450$$

$$= \frac{120}{90} \times 450$$

= ₹ 600

Example 1.15

A dealer allows a discount of 10% and still gains 10%. What is the cost price of the book which is marked at ₹ 220?

Solution

Sugandan used this method:

M.P. = ₹ 220.

Discount = 10% of M.P.

$= \frac{10}{100} \times 220 =$ ₹ 22

S.P. = M.P. – Discount

= 220 – 22 = ₹ 198

Let C.P. be ₹ 100.

Gain = 10% of C. P.

$= \frac{10}{100} \times 100 =$ ₹ 10

S.P. = C.P. + Gain

= 100 + 10

= ₹ 110.

If S.P. is ₹ 110, then C.P. is ₹ 100.

When S.P. is ₹ 198,

C.P. $= \frac{198 \times 100}{110}$

= ₹ 180.

[OR]

Mukundan used the formula method:

Discount = 10%

Gain = 10%

M.P. = ₹ 220

C.P. $= \frac{100 - \text{Discount}\%}{100 + \text{Gain}\%} \times \text{M.P.}$

$= \frac{100 - 10}{100 + 10} \times 220$

$= \frac{90}{110} \times 220 =$ ₹ 180.

Example 1.16

A television set was sold for ₹ 14,400 after giving successive discounts of 10% and 20% respectively. What was the marked price?

Solution

Selling Price = ₹ 14,400

Let the M.P. be ₹ 100.

First discount = $10\% = \frac{10}{100} \times 100 =$ ₹ 10

S.P. after the first discount = $100 - 10 =$ ₹ 90

Second discount = $20\% = \frac{20}{100} \times 90 =$ ₹ 18

Selling Price after the second discount = $90 - 18 =$ ₹ 72

If S.P. is ₹ 72, then M.P. is ₹ 100.

When S.P. is ₹ 14,400,

M.P. = $\frac{14400 \times 100}{72} =$ ₹ 20,000

M.P. = ₹ 20,000.

Example 1.17

A trader buys an article for ₹ 1,200 and marks it 30% above the C.P. He then sells it after allowing a discount of 20%. Find the S.P. and profit percent.

Solution:

Let C.P. of the article be ₹ 100

M.P. = 30% above C.P. = ₹ 130

If C.P. is ₹ 100, then M.P. is ₹ 130.

When C.P. is ₹ 1200, M.P. = $\frac{1200 \times 130}{100} =$ ₹ 1560

Discount = 20% of 1560 = $\frac{20}{100} \times 1560 =$ ₹ 312

S.P. = M.P. – Discount

= $1560 - 312 =$ ₹ 1248

Profit = S.P. – C.P.

= $1248 - 1200 =$ ₹ 48.

$\therefore$ Profit % = $\frac{\text{Profit}}{\text{C.P.}} \times 100$

= $\frac{48}{1200} \times 100 = 4\%$

1.4. Compound Interest

In class VII, we have learnt about Simple Interest and the formula for calculating Simple Interest and Amount. In this chapter, we shall discuss the concept of Compound Interest and the method of calculating Compound Interest and Amount at the end of a certain specified period.

Vinay borrowed ₹ 50,000 from a bank for a fixed time period of 2 years. at the rate of 4% per annum.

Vinay has to pay for the first year,

$$\text{Simple interest} = \frac{P \times n \times r}{100}$$

$$= \frac{50000 \times 1 \times 4}{100} = ₹\ 2{,}000$$

Suppose he fails to pay the simple interest ₹ 2,000 at the end of first year, then the interest ₹ 2,000 is added to the old Principal ₹ 50,000 and now the sum =

P + I = ₹ 52,000 becomes the new Principal for the second year for which the interest is calculated.

Now in the second year he will have to pay an interest of

$$\text{S.I.} = \frac{P \times n \times r}{100}$$

$$= \frac{52000 \times 1 \times 4}{100} = ₹\ 2{,}080$$

Therefore Vinay will have to pay more interest for the second year.

This way of calculating interest is called **Compound Interest.**

Generally in banks, insurance companies, post offices and in other companies which lend money and accept deposits, compound interest is followed to find the interest.

Do you know?

When the interest is paid on the Principal only, it is called **Simple Interest.** But if the interest is paid on the Principal as well as on the accrued interest, it is called **Compound Interest.**

Example 1.21

Ramlal deposited ₹ 8,000 with a finance company for 3 years at an interest of 15% per annum. What is the compound interest that Ramlal gets after 3 years?

Solution

Step 1: Principal for the first year = ₹ 8,000

Interest for the first year $= \frac{P \times n \times r}{100}$

$= \frac{8000 \times 1 \times 15}{100} =$ ₹ 1,200

Amount at the end of **first year** = P + I = 8,000 + 1,200 = ₹ 9,200

Step 2: The amount at the end of the first year becomes the Principal for the second year.

Principal for the second year = ₹ 9,200

Interest for the second year $= \frac{P \times n \times r}{100}$

$= \frac{9200 \times 1 \times 15}{100} =$ ₹ 1,380

Amount at the end of **second year** = P + I = 9,200 + 1,380 = ₹ 10,580

Step 3: The amount at the end of the second year becomes the Principal for the third year.

Principal for the third year = ₹ 10,580

Interest for the third year $= \frac{P \times n \times r}{100}$

$= \frac{10580 \times 1 \times 15}{100} =$ ₹ 1,587

Amount at the end of **third year** = P + I

= 10,580 + 1,587 = ₹ 12,167

Hence, the Compound Interest that Ramlal gets after three years is

A – P = 12,167 – 8,000 = ₹ 4,167.

Deduction of formula for Compound Interest

The above method which we have used for the calculation of Compound Interest is quite lengthy and cumbersome, especially when the period of time is very large. Hence we shall obtain a formula for the computation of Amount and Compound Interest.

If the Principal is P, Rate of interest per annum is r % and the period of time or the number of years is n, then we deduce the compound interest formula as follows:

Step 1 :

$$\text{Principal for the first year} = \text{P}$$

$$\text{Interest for the first year} = \frac{\text{P} \times n \times r}{100}$$

$$= \frac{\text{P} \times 1 \times r}{100} = \frac{\text{P}r}{100}$$

$$\text{Amount at the end of \textbf{first year}} = \text{P} + \text{I}$$

$$= \text{P} + \frac{\text{P}r}{100}$$

$$= \text{P}\left(1 + \frac{r}{100}\right)$$

Step 2 :

$$\text{Principal for the second year} = \text{P}\left(1 + \frac{r}{100}\right)$$

$$\text{Interest for the second year} = \frac{\text{P}\left(1 + \frac{r}{100}\right) \times 1 \times r}{100}$$

(using the S.I.formula)

$$= \text{P}\left(1 + \frac{r}{100}\right) \times \frac{r}{100}$$

$$\text{Amount at the end of \textbf{second year}} = \text{P} + \text{I}$$

$$= \text{P}\left(1 + \frac{r}{100}\right) + \text{P}\left(1 + \frac{r}{100}\right) \times \frac{r}{100}$$

$$= \text{P}\left(1 + \frac{r}{100}\right)\left(1 + \frac{r}{100}\right)$$

$$= \text{P}\left(1 + \frac{r}{100}\right)^2$$

Step 3 : Principal for the third year $= P\left(1+\frac{r}{100}\right)^2$

Interest for the third year $= \dfrac{P\left(1+\frac{r}{100}\right)^2 \times 1 \times r}{100}$

(using the S.I.formula)

$= P\left(1+\frac{r}{100}\right)^2 \times \frac{r}{100}$

Amount at the end of **third year** $= P + I$

$= P\left(1+\frac{r}{100}\right)^2 + P\left(1+\frac{r}{100}\right)^2 \times \frac{r}{100}$

$= P\left(1+\frac{r}{100}\right)^2\left(1+\frac{r}{100}\right)$

$= \mathbf{P\left(1+\frac{r}{100}\right)^3}$

Similarly, Amount at the end of nth year is $A = P\left(1+\frac{r}{100}\right)^n$

and C. I. at the end of 'n' years is given by $A - P$

(i. e.) $C.I. = P\left(1+\frac{r}{100}\right)^n - P$

To Compute Compound Interest

Case 1: Compounded Annually

When the interest is added to the Principal at the end of each year, we say that the interest is compounded annually.

Here $A = P\left(1+\frac{r}{100}\right)^n$ and $C.I. = A - P$

Case 2: Compounded Half - Yearly (Semi - Annually)

When the interest is compounded Half - Yearly, there are two conversion periods in a year each after 6 months. In such situations, the Half - Yearly rate will be half of the annual rate, that is $\left(\frac{r}{2}\right)$.

In this case, $A = P\left[1 + \frac{1}{2}\left(\frac{r}{100}\right)\right]^{2n}$ and C.I. = A – P

Case 3: Compounded Quarterly

When the interest is compounded quarterly, there are four conversion periods in a year and the quarterly rate will be one-fourth of the annual rate, that is $\left(\frac{r}{4}\right)$.

In this case, $A = P\left[1 + \frac{1}{4}\left(\frac{r}{100}\right)\right]^{4n}$ and C.I. = A – P

Case 4: Compounded when time being fraction of a year

When interest is compounded annually but time being a fraction.

In this case, when interest is compounded annually but time being a fraction of a year, say $5\frac{1}{4}$ years, then amount A is given by

$$A = \underbrace{P\left(1 + \frac{r}{100}\right)^5}_{\text{for 5 years}} \underbrace{\left[1 + \frac{1}{4}\left(\frac{r}{100}\right)\right]}_{\text{for } \frac{1}{4} \text{ of year}} \text{ and C.I.} = A - P$$

Example 1.22

Find the C.I. on ₹ 15,625 at 8% p.a. for 3 years compounded annually.

Solution

We know,

$$\text{Amount after 3 years} = P\left(1+\frac{r}{100}\right)^3$$

$$= 15625\left(1+\frac{8}{100}\right)^3$$

$$= 15625\left(1+\frac{2}{25}\right)^3$$

$$= 15625\left(\frac{27}{25}\right)^3$$

$$= 15625 \times \frac{27}{25} \times \frac{27}{25} \times \frac{27}{25}$$

$$= ₹\ 19{,}683$$

$$\text{Now, Compound interest} = A - P = 19{,}683 - 15{,}625$$

$$= ₹\ 4{,}058$$

To find the C.I. when the interest is compounded annually or half-yearly

Let us see what happens to ₹ 100 over a period of one year if an interest is compounded annually or half-yearly.

S.No	Annually	Half-yearly
1	P = ₹ 100 at 10% per annum compounded annually	P = ₹ 100 at 10% per annum compounded half-yearly
2	The time period taken is 1 year	The time period is 6 months or ½ year.
3	$I = \frac{100 \times 10 \times 1}{100} = ₹\ 10$	$I = \frac{100 \times 10 \times \frac{1}{2}}{100} = ₹\ 5$
4	A = 100 + 10 = ₹ 110	A = 100 + 5 = ₹ 105 For the next 6 months, P = ₹ 105 So, $I = \frac{105 \times 10 \times \frac{1}{2}}{100} = ₹\ 5.25$ and A = 105 + 5.25 = ₹ 110.25
5	A = ₹ 110	A = ₹ 110.25

Example 1.23

Find the compound interest on ₹ 1000 at the rate of 10% per annum for 18 months when interest is compounded half-yearly.

Solution

Here, P = ₹ 1000, r = 10% per annum

and $n = 18 \text{ months} = \frac{18}{12} \text{ years} = \frac{3}{2} \text{ years} = 1\frac{1}{2} \text{ years}$

$\therefore$ Amount after 18 months $= P\left[1+\frac{1}{2}\left(\frac{r}{100}\right)\right]^{2n}$

$$= 1000\left[1+\frac{1}{2}\left(\frac{10}{100}\right)\right]^{2\times\frac{3}{2}}$$

$$= 1000\left(1+\frac{10}{200}\right)^{3}$$

$$= 1000\left(\frac{21}{20}\right)^{3}$$

$$= 1000\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}$$

$$= ₹\ 1157.625$$

$$= ₹\ 1157.63$$

C. I. = A – P

= 1157.63 – 1000 = ₹ 157.63

Try these

A sum is taken for one year at 8% p. a. If interest is compounded after every three months, how many times will interest be charged in one year?

Example 1.24

Find the compound interest on ₹ 20,000 at 15% per annum for $2\frac{1}{3}$ years.

Solution

Here, P = ₹ 20,000, r = 15% p. a. and $n = 2\frac{1}{3}$ years.

$$\text{Amount after } 2\frac{1}{3} \text{ years} = A = P\left(1+\frac{r}{100}\right)^2\left[1+\frac{1}{3}\left(\frac{r}{100}\right)\right]$$

$$= 20000\left(1+\frac{15}{100}\right)^2\left[1+\frac{1}{3}\left(\frac{15}{100}\right)\right]$$

$$= 20000\left(1+\frac{3}{20}\right)^2\left(1+\frac{1}{20}\right)$$

$$= 20000\left(\frac{23}{20}\right)^2\left(\frac{21}{20}\right)$$

$$= 20000\times\frac{23}{20}\times\frac{23}{20}\times\frac{21}{20}$$

$$= ₹\ 27,772.50$$

$$\text{C.I.} = A - P$$

$$= 27{,}772.50 - 20{,}000$$

$$= ₹\ 7{,}772.50$$

Inverse Problems on Compound Interest

We have already learnt the formula, $A = P\left(1+\frac{r}{100}\right)^n$,

where four variables A, P, r and n are involved. Out of these four variables, if any three variables are known, then we can calculate the fourth variable.

Example 1.25

At what rate per annum will ₹ 640 amount to ₹ 774.40 in 2 years, when interest is being compounded annually?

Solution:

Given: P = ₹ 640, A = ₹ 774.40, n = 2 years, r = ?

We know,

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$774.40 = 640\left(1+\frac{r}{100}\right)^2$$

$$\frac{774.40}{640} = \left(1+\frac{r}{100}\right)^2$$

$$\frac{77440}{64000} = \left(1+\frac{r}{100}\right)^2$$

$$\frac{121}{100} = \left(1+\frac{r}{100}\right)^2$$

$$\left(\frac{11}{10}\right)^2 = \left(1+\frac{r}{100}\right)^2$$

$$\frac{11}{10} = 1+\frac{r}{100}$$

$$\frac{r}{100} = \frac{11}{10} - 1$$

$$\frac{r}{100} = \frac{11-10}{10}$$

$$\frac{r}{100} = \frac{1}{10}$$

$$r = \frac{100}{10}$$

Rate r = 10% per annum.

Example 1.26

In how much time will a sum of ₹ 1600 amount to ₹ 1852.20 at 5% per annum compound interest.

Solution

Given: P = ₹ 1600, A = ₹ 1852.20, r = 5% per annum, n = ?

We know,

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$1852.20 = 1600\left(1+\frac{5}{100}\right)^n$$

$$\frac{1852.20}{1600} = \left(\frac{105}{100}\right)^n$$

$$\frac{185220}{160000} = \left(\frac{21}{20}\right)^n$$

$$\frac{9261}{8000} = \left(\frac{21}{20}\right)^n$$

$$\left(\frac{21}{20}\right)^3 = \left(\frac{21}{20}\right)^n$$

$$\therefore n = 3 \text{ years}$$

Try these

Find the time period and rate for each of the cases given below :

1. A sum taken for 2 years at 8% p. a. compounded half - yearly.
2. A sum taken for 1½ years at 4% p. a. compounded half - yearly.

1.5 Difference between Simple Interest and Compound Interest

When P is the Principal, n = 2 years and r is the Rate of interest,

Difference between C. I. and S. I. for 2 years = $P\left(\frac{r}{100}\right)^2$

Example 1.27

Find the difference between Simple Interest and Compound Interest for a sum of ₹ 8,000 lent at 10% p. a. in 2 years.

Solution

Here, P = ₹ 8000, n = 2 years, r = 10% p. a.

Difference between Compound Interest and Simple Interest for 2 years = $P\left(\frac{r}{100}\right)^2$

$$= 8000\left(\frac{10}{100}\right)^2$$

$$= 8000\left(\frac{1}{10}\right)^2$$

$$= 8000\times\frac{1}{10}\times\frac{1}{10} = ₹\ 80$$

1.5.1 Appreciation and Depreciation

a) Appreciation

In situations like growth of population, growth of bacteria, increase in the value of an asset, increase in price of certain valuable articles, etc., the following formula is used.

$$A = P\left(1 + \frac{r}{100}\right)^n$$

b) Depreciation

In certain cases where the cost of machines, vehicles, value of some articles, buildings, etc., decreases, the following formula can be used.

$$A = P\left(1 - \frac{r}{100}\right)^n$$

Example 1.28

The population of a village increases at the rate of 7% every year. If the present population is 90,000, what will be the population after 2 years?

Solution

Present population P = 90,000, Rate of increase r = 7%,
Number of years n = 2.

$$\text{The population after '}n\text{' years} = P\left(1 + \frac{r}{100}\right)^n$$

$$\therefore \text{ The population after two years} = 90000\left(1 + \frac{7}{100}\right)^2$$

$$= 90000\left(\frac{107}{100}\right)^2$$

$$= 90000 \times \frac{107}{100} \times \frac{107}{100}$$

$$= 103041$$

$$\text{The population after two years} = 1{,}03{,}041$$

Example 1.29

The value of a machine depreciates by 5% each year. A man pays ₹ 30,000 for the machine. Find its value after three years.

Solution

Present value of the machine P = ₹ 30,000, Rate of depreciation $r = 5\%$,

$$\text{Number of years } n = 3$$

$$\text{The value of the machine after '}n\text{' years} = P\left(1-\frac{r}{100}\right)^n$$

$$\therefore \text{ The value of the machine after three years} = 30000\left(1-\frac{5}{100}\right)^3$$

$$= 30000\left(\frac{95}{100}\right)^3$$

$$= 30000\times\frac{95}{100}\times\frac{95}{100}\times\frac{95}{100}$$

$$= 25721.25$$

$$\text{The value of the machine after three years} = ₹\ 25{,}721.25$$

Example 1.30

The population of a village has a constant growth of 5% every year. If its present population is 1,04,832, what was the population two years ago?

Solution

Let P be the population two years ago.

$$\therefore\ P\left(1+\frac{5}{100}\right)^2 = 104832$$

$$P\left(\frac{105}{100}\right)^2 = 104832$$

$$P\times\frac{105}{100}\times\frac{105}{100} = 104832$$

$$P = \frac{104832\times100\times100}{105\times105}$$

$$= 95085.71$$

$$= 95{,}086 \text{ (rounding off to the nearest whole number)}$$

$\therefore$ Two years ago the population was 95,086.

To find the formula for calculating interest and the maturity amount for R.D :

Let r % be the rate of interest paid and 'P' be the monthly instalment paid for 'n' months.

$$\text{Interest} = \frac{PNr}{100}, \text{ where } N = \frac{1}{12}\left[\frac{n(n+1)}{2}\right] \text{ years}$$

$$\text{Total Amount due at maturity is } A = Pn + \frac{PNr}{100}$$

Example 1.31

Tharun makes a deposit of Rupees two lakhs in a bank for 5 years. If the rate of interest is 8% per annum, find the maturity value.

Solution

Principal deposited P = ₹ 2,00,000, n = 5 years, r = 8% p. a.

$$\text{Interest} = \frac{Pnr}{100} = 200000 \times 5 \times \frac{8}{100}$$
$$= ₹\ 80,000$$

$$\therefore \text{ Maturity value after 5 years} = 2,00,000 + 80,000 = ₹\ 2,80,000.$$

Example 1.32

Vaideesh deposits ₹ 500 at the beginning of every month for 5 years in a post office. If the rate of interest is 7.5%, find the amount he will receive at the end of 5 years.

Solution

Amount deposited every month, P = ₹ 500

Number of months, n = 5 × 12 = 60 months

Rate of interest, r = $7\frac{1}{2}\% = \frac{15}{2}\%$

Total deposit made = Pn = 500 × 60

= ₹ 30,000

Period for recurring deposit, N = $\frac{1}{12}\left[\frac{n(n+1)}{2}\right]$ years

$=\frac{1}{24}\times 60\times 61 = \frac{305}{2}$ years

$$\text{Interest, I} = \frac{PNr}{100}$$

$$= 500 \times \frac{305}{2} \times \frac{15}{2 \times 100}$$

$$= ₹\ 5,718.75$$

$$\text{Total amount due} = Pn + \frac{PNr}{100}$$

$$= 30,000 + 5,718.75$$

$$= ₹\ 35,718.75$$

Example 1.33

Vishal deposited ₹ 200 per month for 5 years in a recurring deposit account in a post office. If he received ₹ 13,830 find the rate of interest.

Solution

Maturity Amount, A = ₹ 13,830, P = ₹ 200, $n = 5 \times 12 = 60$ months

$$\text{Period, N} = \frac{1}{12}\left[\frac{n(n+1)}{2}\right] \text{ years}$$

$$= \frac{1}{12} \times 60 \times \frac{61}{2} = \frac{305}{2} \text{ years}$$

$$\text{Amount Deposited} = Pn = 200 \times 60 = ₹\ 12,000$$

$$\text{Maturity Amount} = Pn + \frac{PNr}{100}$$

$$13830 = 12000 + 200 \times \frac{305}{2} \times \frac{r}{100}$$

$$13830 - 12000 = 305 \times r$$

$$1830 = 305 \times r$$

$$\therefore r = \frac{1830}{305} = 6\%$$

Equated Monthly Instalment (E.M.I.)

Equated Monthly Instalment is also as equivalent as the instalment scheme but with a dimnishing concept. We have to repay the cost of things with the interest along with certain charges. The total amount should be divided by the period of months. The amount thus arrived is known as Equated Monthly Instalment.

$$\text{E.M.I} = \frac{\text{Principal + Interest}}{\text{Number of months}}$$

Different schemes of Hire purchase and Instalment scheme

1. 0% interest scheme: Companies take processing charge and 4 or 5 months instalments in advance.

2. 100% Finance: Companies add interest and the processing charges to the cost price.

3. Discount Sale: To promote sales, discount is given in the instalment schemes.

4. Initial Payment: A certain part of the price of the article is paid towards the purchase in advance. It is also known as Cash down payment.

1.7 Compound Variation

In the earlier classes we have already learnt about Direct and Inverse Variation. Let us recall them.

Direct Variation

If two quantities are such that an increase or decrease in one leads to a corresponding increase or decrease in the other, we say they vary directly or the variation is Direct.

Examples for Direct Variation:

1. Distance and Time are in Direct Variation, because more the distance travelled, the time taken will be more(if speed remains the same).
2. Principal and Interest are in Direct Variation, because if the Principal is more the interest earned will also be more.
3. Purchase of Articles and the amount spent are in Direct Variation, because purchase of more articles will cost more money.

Indirect Variation or Inverse Variation:

If two quantities are such that an increase or decrease in one leads to a corresponding decrease or increase in the other, we say they vary indirectly or the variation is inverse.

Examples for Indirect Variation:

1. Work and time are in Inverse Variation, because more the number of the workers, lesser will be the time required to complete a job.
2. Speed and time are in Inverse Variation, because higher the speed, the lower is the time taken to cover a distance.
3. Population and quantity of food are in Inverse Variation, because if the population increases the food availability decreases.

Compound Variation

Certain problems involve a chain of two or more variations, which is called as Compound Variation.

The different possibilities of variations involving two variations are shown in the following table:

Variation I	Variation II
Direct	Direct
Inverse	Inverse
Direct	Inverse
Inverse	Direct

Let us work out some problems to illustrate compound variation.

Example 1.36

If 20 men can build a wall 112 meters long in 6 days, what length of a similar wall can be built by 25 men in 3 days?

Solution:

Method 1: The problem involves set of 3 variables, namely- Number of men, Number of days and length of the wall.

Number of Men	Number of days	Length of the wall in metres
20	6	112
25	3	x

Therefore, the proportion is $20 : 25 :: 112 : x$ ...... (1)

Step 2: Consider the number of days and the length of the wall. As the number of days decreases from 6 to 3, the length of the wall also decreases. So, it is in Direct Variation.

Therefore, the proportion is $6 : 3 :: 112 : x$ ..... (2)

Combining (1) and (2), we can write

$$\left.\begin{array}{r}20:25\\6:3\end{array}\right\}::112:x$$

We know, **Product of Extremes = Product of Means.**

Extremes		Means		Extremes
20	:	25 ::112	:	x
6	:	3		

So, $20 \times 6 \times x = 25 \times 3 \times 112$

$$x = \frac{25 \times 3 \times 112}{20 \times 6} = 70 \text{ meters.}$$

Method 2

Number of Men	Number of days	Length of the wall in metres
20	6	112
25	3	x

Step 1: Consider the number of men and length of the wall. As the number of men increases from 20 to 25, the length of the wall also increases. It is in direct variation.

The multiplying factor $= \frac{25}{20}$

Step 2: Consider the number of days and the length of the wall. As the number of days decreases from 6 to 3, the length of the wall also decreases. It is in direct variation.

The multiplying factor $= \frac{3}{6}$.

$$\therefore x = \frac{25}{20} \times \frac{3}{6} \times 112 = 70 \text{ meters}$$

Example 1.37

Six men working 10 hours a day can do a piece of work in 24 days. In how many days will 9 men working for 8 hours a day do the same work?

Solution

Method 1: The problem involves 3 sets of variables, namely - Number of men, Working hours per day and Number of days.

Number of Men	Number of hours per day	Number of days
6	10	24
9	8	x

Step 1: Consider the number of men and the number of days. As the number of men increases from 6 to 9, the number of days decreases. So it is in Inverse Variation.

Therefore the proportion is $9 : 6 :: 24 : x$ ..... (1)

Step 2: Consider the number of hours worked per day and the number of days. As the number of hours working per day decreases from 10 to 8, the number of days increases. So it is inverse variation.

Therefore the proportion is $8 : 10 :: 24 : x$ ..... (2)

Combining (1) and (2), we can write as

$$\left.\begin{matrix} 9:6 \\ 8:10 \end{matrix}\right\} :: 24 : x$$

We know, **Product of extremes = Product of Means.**

Extremes		Means		Extremes
9	:	6 : : 24	:	x
8	:	10		

So, $9 \times 8 \times x = 6 \times 10 \times 24$

$$x = \frac{6 \times 10 \times 24}{9 \times 8} = 20 \text{ days}$$

Note: 1. Denote the Direct variation as ↓ (Downward arrow)
2. Denote the Indirect variation as ↑ (Upward arrow)
3. Multiplying Factors can be written based on the arrows. Take the number on the head of the arrow in the numerator and the number on the tail of the arrow in the denominator.

For method two, use the instructions given in the note above .

Method 2 : (Using arrow marks)

Number of Men	Number of hours per day	Number of days
6	10	24
9	8	x

Step 1 : Consider men and days. As the number of men increases from 6 to 9, the number of days decreases. It is in inverse variation.

The multiplying factor = $\frac{6}{9}$

Step 2 : Consider the number of hours per day and the number of days. As the number of hours per day decreases from 10 to 8, the number of days increases. It is also in inverse variation.

The multiplying factor = $\frac{10}{8}$

$$\therefore x = \frac{6}{9} \times \frac{10}{8} \times 24 = 20 \text{ days.}$$

1.8 Time and Work

When we have to compare the work of several persons, it is necessary to ascertain the amount of work each person can complete in one day. As time and work are of inverse variation and if more people are joined to do a work, the work will be completed within a shorter time.

In solving problems here, the following points should be remembered:

1. If a man finishes total work in 'n' days, then in one day he does '$\frac{1}{n}$' of the total work. For example, if a man finishes a work in 4 days, then in one day he does $\frac{1}{4}$ of the work.
2. If the quantity of work done by a man in one day is given, then the total number of days taken to finish the work = 1/(one day's work). For example, if a man does $\frac{1}{10}$ of the work in 1 day, then the number of days taken to finish the work

$$= \frac{1}{\left(\frac{1}{10}\right)} = 1 \times \frac{10}{1} = 10 \text{ days.}$$

Example 1.38

A can do a piece of work in 20 days and B can do it in 30 days. How long will they take to do the work together?

Solution

Work done by A in 1 day = $\frac{1}{20}$, Work done by B in 1 day = $\frac{1}{30}$

$$\text{Work done by A and B in 1 day} = \frac{1}{20} + \frac{1}{30}$$

$$= \frac{3+2}{60} = \frac{5}{60} = \frac{1}{12} \text{ of the work}$$

Total number of days required to finish the work by A and B = $\frac{1}{1/12} = 12$ days.

Example 1.39

A and B together can do a piece of work in 8 days, but A alone can do it 12 days. How many days would B alone take to do the same work?

Solution

$$\text{Work done by A and B together in 1 day} = \frac{1}{8} \text{ of the work}$$

$$\text{Work done by A in 1 day} = \frac{1}{12} \text{ of the work}$$

$$\text{Work done by B in 1 day} = \frac{1}{8} - \frac{1}{12} = \frac{3-2}{24} = \frac{1}{24}$$

Number of days taken by B alone to do the same work = $\frac{1}{1/24} = 24$ days.

Example 1.40

Two persons A and B are engaged in a work. A can do a piece of work in 12 days and B can do the same work in 20 days. They work together for 3 days and then A goes away. In how many days will B finish the work?

Try these

While A, B and C working individually can complete a job in 20,5,4 days respectively. If all join together and work, find in how many days they will finish the job?

Solution

$$\text{Work done by A in 1 day} = \frac{1}{12}$$

$$\text{Work done by B in 1 day} = \frac{1}{20}$$

$$\text{Work done by A and B together in 1 day} = \frac{1}{12}+\frac{1}{20}$$

$$= \frac{5+3}{60} = \frac{8}{60} = \frac{2}{15}$$

$$\text{Work done by A and B together in 3 days} = \frac{2}{15}\times 3 = \frac{2}{5}$$

$$\text{Remaining Work} = 1-\frac{2}{5} = \frac{3}{5}$$

$$\text{Number of days taken by B to finish the remaining work} = \frac{\frac{3}{5}}{\frac{1}{20}} = \frac{3}{5}\times\frac{20}{1}$$

$$= 12 \text{ days.}$$

Example 1.41

A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. In how many days will they finish it together and separately?

Solution

$$\text{Work done by A and B in 1 day} = \frac{1}{12}$$

$$\text{Work done by B and C in 1 day} = \frac{1}{15}$$

$$\text{Work done by C and A in 1 day} = \frac{1}{20}$$

$$\text{Work done by (A+B)+(B+C)+(C+A) in 1 day} = \frac{1}{12}+\frac{1}{15}+\frac{1}{20}$$

$$\text{Work done by } (2A+2B+2C) \text{ in 1 day} = \frac{5+4+3}{60}$$

$$\text{Work done by } 2(A+B+C) \text{ in 1 day} = \frac{12}{60}$$

$$\text{Work done by A, B and C together in 1 day} = \frac{1}{2}\times\frac{12}{60} = \frac{1}{10}$$

$\therefore$ A,B and C will finish the work in 10 days.

Work done by A in 1 day

$$\text{(i.e.)} \quad [(A+B+C)\text{'s work} - (B+C)\text{'s work}] = \frac{1}{10} - \frac{1}{15} = \frac{3-2}{30} = \frac{1}{30}$$

$\therefore$ A will finish the work in 30 days.

Workdone by B in 1 day

$$\text{(i.e.)} \quad [(A+B+C)\text{'s work} - (C+A)\text{'s work}] = \frac{1}{10} - \frac{1}{20} = \frac{2-1}{20} = \frac{1}{20}$$

$\therefore$ B will finish the work in 20 days.

Work done by C in 1 day

$$\text{(i.e.)} \quad [(A+B+C)\text{'s work} - (A+B)\text{'s work}] = \frac{1}{10} - \frac{1}{12} = \frac{6-5}{60} = \frac{1}{60}$$

$\therefore$ C will finish the work in 60 days.

Example 1.42

A can do a piece of work in 10 days and B can do it in 15 days. How much does each of them get if they finish the work and earn ₹ 1500?

Solution

$$\text{Work done by A in 1 day} = \frac{1}{10}$$

$$\text{Work done by B in 1 day} = \frac{1}{15}$$

$$\text{Ratio of their work} = \frac{1}{10} : \frac{1}{15} = 3 : 2$$

$$\text{Total Share} = ₹\ 1500$$

$$\text{A's share} = \frac{3}{5}\times 1500 = ₹\ 900$$

$$\text{B's share} = \frac{2}{5}\times 1500 = ₹\ 600$$

Example 1.43

Two taps can fill a tank in 30 minutes and 40 minutes. Another tap can empty it in 24 minutes. If the tank is empty and all the three taps are kept open, in how much time the tank will be filled?

Solution

Quantity of water filled by the first tap in one minute $= \frac{1}{30}$

Quantity of water filled by the second tap in one minute $= \frac{1}{40}$

Quantity of water emptied by the third tap in one minute $= \frac{1}{24}$

Quantity of water filled in one minute, when all the 3 taps are opened $\left.\right\} = \frac{1}{30} + \frac{1}{40} - \frac{1}{24}$

$$= \frac{4+3-5}{120} = \frac{7-5}{120}$$

$$= \frac{2}{120} = \frac{1}{60}$$

Time taken to fill the tank $= \frac{1}{1/60} = 60$ minutes

$= 1$ hour

Abbreviation: C.P. = Cost Price, S.P. = Selling Price, M.P. = Marked Price, P = Principal, r = Rate of interest, n = time period, A = Amount, C. I. = Compound Interest.

- Percent means per hundred. A fraction with its denominator 100 is called a percent.
- In case of profit, we have

  Profit $=$ S.P. $-$ C.P. ; Profit percent $= \frac{\text{Profit}}{\text{C.P}} \times 100$

  S.P. $= \left(\frac{100 + \text{Profit\%}}{100}\right) \times$ C.P.; C.P. $= \left(\frac{100}{100 + \text{Profit\%}}\right) \times$ S.P.
- In case of Loss, we have

  Loss $=$ C.P. $-$ S.P.; Loss percent $= \frac{\text{Loss}}{\text{C.P.}} \times 100$

  S. P. $= \left(\frac{100 - \text{Loss\%}}{100}\right) \times$ C.P.; C.P. $= \left(\frac{100}{100 - \text{Loss\%}}\right) \times$ S.P.
- Discount is the reduction given on the Marked Price.
- Selling Price is the price payable after reducing the Discount from the Marked Price.
- Discount = M.P. – S.P.
- M.P. $= \frac{100}{100 - \text{Discount\%}} \times$ S.P.; S.P. $= \frac{100 - \text{Discount\%}}{100} \times$ M.P.
- C.P. $= \frac{100 - \text{Discount\%}}{100 + \text{Profit\%}} \times$ M.P.; M.P. $= \frac{100 + \text{Profit\%}}{100 - \text{Discount\%}} \times$ C.P.
- Discount Percent $= \frac{\text{Discount}}{\text{M.P.}} \times 100.$
- When the interest is

  (i) compounded annually, $A = P\left(1 + \frac{r}{100}\right)^n$

  (ii) compounded half - yearly, $A = P\left[1 + \frac{1}{2}\left(\frac{r}{100}\right)\right]^{2n}$

  (iii) compounded quarterly, $A = P\left[1 + \frac{1}{4}\left(\frac{r}{100}\right)\right]^{4n}$
- Appreciation, $A = P\left(1 + \frac{r}{100}\right)^n$; Depreciation, $A = P\left(1 - \frac{r}{100}\right)^n$
- The difference between C. I. and S. I. for 2 years $= P\left(\frac{r}{100}\right)^2$
- One day's work of A $= \frac{1}{\text{Number of days taken by A}}$
- Work completed in 'x' days = One day's work x x

Pythagorean Triplets.

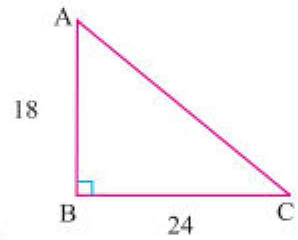
Example 2.1

In $\Delta ABC, \angle B = 90^\circ$, AB = 18cm and BC = 24cm. Calculate the length of AC.

Solution

By Pythagoras Theorem,

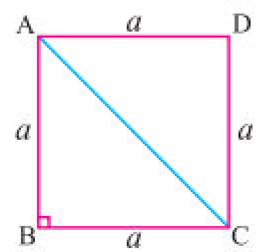
$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 18^2 + 24^2 \\ &= 324 + 576 \\ &= 900 \\ \therefore AC &= \sqrt{900} = 30\text{cm} \end{aligned}$$



Example 2.2

A square has the perimeter 40cm. What is the sum of the diagonals?

Solution



Let '*a*' be the length of the side of the square. AC is a diagonal.

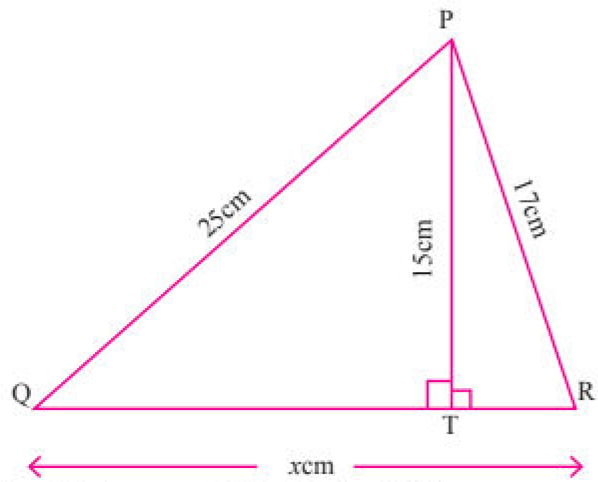
$$\begin{aligned} \text{Perimeter of square ABCD} &= 4\,a \text{ units} \\ 4a &= 40\text{cm [given]} \\ a &= \frac{40}{4} = 10\text{cm} \end{aligned}$$

We know that in a square each angle is 90° and the diagonals are equal.

$$\begin{aligned} \text{In } \Delta ABC,\ AC^2 &= AB^2 + BC^2 \\ &= 10^2 + 10^2 = 100 + 100 = 200 \\ \therefore AC &= \sqrt{200} \\ &= \sqrt{2 \times 100} = 10\sqrt{2} \\ &= 10 \times 1.414 = 14.14\text{cm} \\ \text{Diagonal AC} &= \text{Diagonal BD} \\ \text{Hence, Sum of the diagonals} &= 14.14 + 14.14 = 28.28\text{cm.} \end{aligned}$$

Example 2.3

From the figure PT is an altitude of the triangle PQR in which PQ = 25cm, PR = 17cm and PT = 15 cm. If QR = x cm. Calculate x.



Solution From the figure, we have QR = QT + TR.

To find : QT and TR.

In the right angled triangle PTQ , $\angle PTQ = 90°$ [PT is attitude]

By Pythagoras Theorem, $PQ^2 = PT^2 + QT^2$ $\quad \therefore PQ^2 - PT^2 = QT^2$

$$\therefore QT^2 = 25^2 - 15^2 = 625 - 225 = 400$$

$$QT = \sqrt{400} = 20 \text{ cm} \qquad .....(1)$$

Similarly, in the right angled triangle PTR,

by Pythagoras Theorem, $PR^2 = PT^2 + TR^2$

$$\therefore TR^2 = PR^2 - PT^2$$

$$= 17^2 - 15^2$$

$$= 289 - 225 = 64$$

$$TR = \sqrt{64} = 8 \text{ cm} \qquad ..... (2)$$

Form (1) and (2) $\quad QR = QT + TR = 20 + 8 = 28\text{cm}.$

Example 2.4

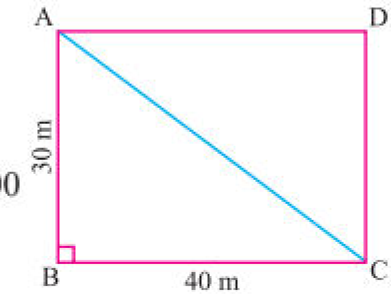
A rectangular field is of dimension 40m by 30m. What distance is saved by walking diagonally across the field?

Solution

Given: ABCD is a rectangular field of Length = 40m, Breadth = 30m, $\angle B = 90^\circ$

In the right angled triangle ABC,

By Pythagoras Theorem,



$$AC^2 = AB^2 + BC^2$$
$$= 30^2 + 40^2 = 900 + 1600$$
$$= 2500$$
$$\therefore AC = \sqrt{2500} = 50 \text{ m}$$

Distance from A to C through B is

$$= 30 + 40 = 70 \text{ m}$$

$$\text{Distance saved} = 70 - 50 = 20 \text{ m.}$$

Diameter

A diameter is a chord that passes through the centre of the circle and diameter is the longest chord of a circle.

In the figure, AOB is diameter of the circle.

O is the mid point of AB and OA= OB = radius of the circle

Hence, Diameter = 2 × radius (or) Radius = (diameter) ÷ 2

Note : (i) The mid-point of every diameter of the circle is the centre of the circle.

(ii) The diameters of a circle are concurrent and the point of concurrency is the centre of the circle.

- **Centroid :** Point of concurrency of the three Medians.
- **Orthocentre :** Point of concurrency of the three Altitudes.
- **Incentre :** Point of concurrency of the three Angle Bisectors.
- **Circumcentre :** Point of concurrency of the Perpendicular Bisectors of the three sides.
- **Circle :** A circle is the set of all points in a plane at a constant distance from a fixed point in that plane .
- **Chord :** A chord is a line segment with its end points lying on a circle.
- **Diameter :** A diameter is a chord that passes through the centre of the circle.
- A line passing through a circle and intersecting the circle at two points is called the secant of the circle.
- Tangent is a line that touches a circle at exactly one point, and the point is known as point of contact.
- **Segment of a circle :** A chord of a circle divides the circular region into two parts.
- **Sector of a circle :** The circular region enclosed by an arc of a circle and the two radii at its end points is known as Sector of a circle.

3.5 Measures of Central Tendency

Even after tabulating the collected mass of data, we get only a hazy general picture of the distribution. To obtain a more clear picture, it would be ideal if we can describe the whole mass of data by a single number or representative number. To get more information about the tendency of the data to deviate about a particular value, there are certain measures which characterise the entire data. These measures are called the **Measures of Central Tendency**. Such measures are

(i) Arithmetic Mean, (ii) Median and (iii) Mode

3.5.1 Arithmetic Mean (A.M)

The arithmetic mean is the ratio of the sum of all the observations to the total number of observations.

3.5.1. (a) Arithmetic mean for ungrouped data

If there are n observations $x_1, x_2, x_3, \cdots, x_n$ for the variable x then their arithmetic mean is denoted by $\overline{x}$ and it is given by $\overline{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$.

In Mathematics, the symbol in Greek letter Σ, is called **Sigma.** This notation is used to represent the summation. With this symbol, the sum of $x_1, x_2, x_3, \cdots, x_n$ is denoted as $\sum_{i=1}^{n} x_i$ or simply as Σx_i. Then we have $\overline{x} = \frac{\Sigma x_i}{n}$.

Note: Arithmetic mean is also known as **Average** or **Mean**.

More about Notation : Σ

$$\sum_{k=1}^{3} k = 1 + 2 + 3 = 6$$

$$\sum_{n=3}^{6} n = 3 + 4 + 5 + 6 = 18$$

$$\sum_{n=2}^{4} 2n = 2 \times 2 + 2 \times 3 + 2 \times 4 = 18$$

$$\begin{aligned} \sum_{k=1}^{3} 5 &= \sum_{k=1}^{3} 5 \times k^0 \\ &= 5 \times 1^0 + 5 \times 2^0 + 5 \times 3^0 \\ &= 5 + 5 + 5 = 15 \end{aligned}$$

$$\sum_{K=2}^{4} (k-1) = (2-1) + (3-1) + (4-1) = 6$$

Example 3.12

The marks obtained by 10 students in a test are 15, 75, 33, 67, 76, 54, 39, 12, 78, 11. Find the arithmetic mean.

Solution

Here, the number of observations, $n = 10$

$$\text{A. M} = \overline{x} = \frac{15 + 75 + 33 + 67 + 76 + 54 + 39 + 12 + 78 + 11}{10}$$

$$\overline{x} = \frac{460}{10} = 46.$$

Example 3.13

If the average of the values 9, 6, 7, 8, 5 and x is 8. Find the value of x.

Solution

Here, the given values are 9, 6, 7, 8, 5 and x, also $n = 6$.

$$\text{By formula, A.M.} = \overline{x} = \frac{9 + 6 + 7 + 8 + 5 + x}{6} = \frac{35 + x}{6}$$

$$\text{By data, } \overline{x} = 8$$

$$\text{So, } \frac{35 + x}{6} = 8$$

$$\text{i.e. } 35 + x = 48$$

$$x = 48 - 35 = 13.$$

Example 3.14

The average height of 10 students in a class was calculated as 166 cm. On verification it was found that one reading was wrongly recorded as 160 cm instead of 150 cm. Find the correct mean height.

Solution

Here, $\overline{x} = 166$ cm and $n = 10$

We have $\overline{x} = \frac{\Sigma x}{n} = \frac{\Sigma x}{10}$

i.e. $166 = \frac{\Sigma x}{10}$ or $\Sigma x = 1660$

The incorrect $\Sigma x = 1660$

The correct Σx = incorrect Σx – the wrong value + correct value

$= 1660 - 160 + 150 = 1650$

Hence, the correct A.M. $= \frac{1650}{10} = 165$ cm.

Example 3.15

Calculate the Arithmetic mean of the following data by direct method

x	5	10	15	20	25	30
f	4	5	7	4	3	2

Solution

x	f	fx
5	4	20
10	5	50
15	7	105
20	4	80
25	3	75
30	2	60
Total	N = 25	$\Sigma fx = 390$

Arithmetic Mean, $\overline{x} = \frac{\Sigma f x}{N}$

$= \frac{390}{25} = 15.6$.

3.5.3 Median

Another measure of central tendency is the **Median**.

3.5.3 (a) To find Median for ungrouped data

The median is calculated as follows:

(i) Suppose there are an **odd number** of observations, write them in ascending or descending order. Then the middle term is the Median.

For example: Consider the five observations 33, 35, 39, 40, 43. The middle most value of these observation is 39. It is the Median of these observation.

(ii) Suppose there are an **even number** of observations, write them in ascending or descending order. Then the average of the two middle terms is the Median.

For example, the median of 33, 35, 39, 40, 43, 48 is $\frac{39+40}{2} = 39.5$.

Example 3.17

Find the median of 17, 15, 9, 13, 21, 7, 32.

Solution

Arrange the values in the ascending order as 7, 9, 13, 15, 17, 21,32,

$$\text{Here, } n = 7 \text{ (odd number)}$$

$$\text{Therefore, Median} = \text{Middle value}$$

$$= \left(\frac{n+1}{2}\right)^{th} \text{value} = \left(\frac{7+1}{2}\right)^{th} \text{value} = 4\text{th value.}$$

Hence, the median is 15.

Example 3.18

A cricket player has taken the runs 13, 28, 61, 70, 4, 11, 33, 0, 71, 92. Find the median.

Solution

Arrange the runs in ascending order as 0, 4, 11, 13, 28, 33, 61, 70, 71, 92.

Here $n = 10$ (even number).

There are two middle values 28 and 33.

$$\therefore \text{Median} = \text{Average of the two middle values}$$

$$= \frac{28+33}{2} = \frac{61}{2} = 30.5 .$$

3.5.3 (b) To find Median for grouped data

Cumulative frequency

Cumulative frequency of a class is nothing but the total frequency upto that class.

Example 3.19

Find the median for marks of 50 students

Marks	20	27	34	43	58	65	89
Number of students	2	4	6	11	12	8	7

Solution

Marks (x)	Number of students (f)	Cumulative frequency
20	2	2
27	4	$(2+4=)$ 6
34	6	$(6+6=)$ 12
43	11	$(11+12=)$ 23
58	**12**	**$(23+12=)$ 35**
65	8	$(35+8=)$ 43
89	7	$(43+7=)$ 50

Here, the total frequency, $N = \Sigma f = 50$

$$\therefore \frac{N}{2} = \frac{50}{2} = 25.$$

The median is $\left(\frac{N}{2}\right)^{th}$ value = 25th value.

Now, **25th value occurs in the cummulative frequency 35, whose corresponding marks is 58.**

Hence, the median = 58.

3.5.4 Mode

Mode is also a measure of central tendency.

The Mode can be calculated as follows:

3.5.4 (a) To find Mode for ungrouped data (Discrete data)

If a set of individual observations are given, then the Mode is the value which occurs most often.

Example 3.20

Find the mode of 2, 4, 5, 2, 1, 2, 3, 4, 4, 6, 2.

Solution

In the above example the number 2 occurs maximum number of times.

ie, 4 times. Hence mode = 2.

Example 3.21

Find the mode of 22, 25, 21, 22, 29, 25, 34, 37, 30, 22, 29, 25.

Solution

Here 22 occurs 3 times and 25 also occurs 3 times

$\therefore$ Both 22 and 25 are the modes for this data. We observe that there are two modes for the given data.

Example 3.22

Find the mode of 15, 25, 35, 45, 55, 65,

Solution

Each value occurs exactly one time in the series. Hence there is no mode for this data.

3.5.4 (b) To find Mode for grouped data (Frequency distribution)

If the data are arranged in the form of a frequency table, the class corresponding to the maximum frequency is called the modal class. The value of the variate of the modal class is the **mode**.

Example: 3.23

Find the mode for the following frequency table

Wages (₹)	250	300	350	400	450	500
Number of workers	10	15	16	12	11	13

Solution

Wages (₹)	Number of workers
250	10
300	15
350	**16**
400	12
450	11
500	13

We observe from the above table that the maximum frequency is 16. **The value of the variate (wage) corresponding to the maximum frequency 16 is 350.** This is the mode of the given data.

Samacheer Kalvi Maths

9th Std

Remark If a is a rational number and $\sqrt{b}$ is an irrational number then

(i) $a+\sqrt{b}$ is irrational (ii) $a-\sqrt{b}$ is irrational

(iii) $a\sqrt{b}$ is irrational (iv) $\frac{a}{\sqrt{b}}$ is irrational (v) $\frac{\sqrt{b}}{a}$ is irrational

For example, (i) $2+\sqrt{3}$ is irrational (ii) $2-\sqrt{3}$ is irrational

(iii) $2\sqrt{3}$ is irrational (iv) $\frac{2}{\sqrt{3}}$ is irrational

2.4.4 Square Root of Real Numbers

Let $a>0$ be a real number. Then $\sqrt{a}=b$ means $b^2=a$ and $b>0$.

2 is a square root of 4 because $2\times 2=4$, but -2 is also a square root of 4 because $(-2)\times(-2)=4$. To avoid confusion between these two we define the symbol $\sqrt{\ }$, to mean the principal or positive square root.

Let us now mention some useful identities relating to square roots.

Let a and b be positive real numbers. Then	
1	$\sqrt{ab}=\sqrt{a}\sqrt{b}$
2	$\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$
3	$(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=a-b$
4	$(a+\sqrt{b})(a-\sqrt{b})=a^2-b$
5	$(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{d})=\sqrt{ac}+\sqrt{ad}+\sqrt{bc}+\sqrt{bd}$
6	$(\sqrt{a}+\sqrt{b})^2=a+b+2\sqrt{ab}$

1.2 Algebraic Identities

Key Concept **Algebraic Identities**

An identity is an equality that remains true regardless of the values of any variables that appear within it.

We have learnt the following identities in class VIII. Using these identities let us solve some problems and extend the identities to trinomials and third degree expansions.

$(a+b)^2 \equiv a^2 + 2ab + b^2$ $\quad$ $(a+b)(a-b) \equiv a^2 - b^2$

$(a-b)^2 \equiv a^2 - 2ab + b^2$ $\quad$ $(x+a)(x+b) \equiv x^2 + (a+b)x + ab$

Example 1.1

Expand the following using identities

(i) $(2a+3b)^2$ (ii) $(3x-4y)^2$ (iii) $(4x+5y)(4x-5y)$ (iv) $(y+7)(y+5)$

Solution

(i) $(2a+3b)^2 = (2a)^2 + 2(2a)(3b) + (3b)^2$

$= 4a^2 + 12ab + 9b^2$

(ii) $(3x-4y)^2 = (3x)^2 - 2(3x)(4y) + (4y)^2$

$= 9x^2 - 24xy + 16y^2$

(iii) $(4x+5y)(4x-5y) = (4x)^2 - (5y)^2$

$= 16x^2 - 25y^2$

(iv) $(y+7)(y+5) = y^2 + (7+5)y + (7)(5)$

$= y^2 + 12y + 35$

1.2.1 Expansion of the Trinomial $(x \pm y \pm z)^2$

$$\begin{aligned}(x+y+z)^2 &= (x+y+z)(x+y+z)\\ &= x(x+y+z) + y(x+y+z) + z(x+y+z)\\ &= x^2 + xy + xz + yx + y^2 + yz + zx + zy + z^2\\ &= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\end{aligned}$$

$$(x+y+z)^2 \equiv x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

(ii) $(x-y+z)^2 = [x+(-y)+z]^2$

$= x^2 + (-y)^2 + z^2 + 2(x)(-y) + 2(-y)(z) + 2(z)(x)$

$$= x^2 + y^2 + z^2 - 2xy - 2yz + 2zx$$

$$(x - y + z)^2 \equiv x^2 + y^2 + z^2 - 2xy - 2yz + 2zx$$

In the same manner we get the expansion for the following

(iii) $(x + y - z)^2 \equiv x^2 + y^2 + z^2 + 2xy - 2yz - 2zx$

(iv) $(x - y - z)^2 \equiv x^2 + y^2 + z^2 - 2xy + 2yz - 2zx$

Example 1.2

Expand (i) $(2x + 3y + 5z)^2$ (ii) $(3a - 7b + 4c)^2$ (iii) $(3p + 5q - 2r)^2$

(iv) $(7l - 9m - 6n)^2$

Solution

(i) $(2x + 3y + 5z)^2 = (2x)^2 + (3y)^2 + (5z)^2 + 2(2x)(3y) + 2(3y)(5z) + 2(5z)(2x)$

$$= 4x^2 + 9y^2 + 25z^2 + 12xy + 30yz + 20zx$$

(ii) $(3a - 7b + 4c)^2$

$$= (3a)^2 + (-7b)^2 + (4c)^2 + 2(3a)(-7b) + 2(-7b)(4c) + 2(4c)(3a)$$

$$= 9a^2 + 49b^2 + 16c^2 - 42ab - 56bc + 24ca$$

(iii) $(3p + 5q - 2r)^2$

$$= (3p)^2 + (5q)^2 + (-2r)^2 + 2(3p)(5q) + 2(5q)(-2r) + 2(-2r)(3p)$$

$$= 9p^2 + 25q^2 + 4r^2 + 30pq - 20qr - 12rp$$

(iv) $(7l - 9m - 6n)^2$

$$= (7l)^2 + (-9m)^2 + (-6n)^2 + 2(7l)(-9m) + 2(-9m)(-6n) + 2(-6n)(7l)$$

$$= 49l^2 + 81m^2 + 36n^2 - 126lm + 108mn - 84nl$$

1.2.2 Identities Involving Product of Binomials $(x+a)(x+b)(x+c)$

$$(x+a)(x+b)(x+c) = [(x+a)(x+b)](x+c)$$

$$= [x^2 + (a+b)x + ab](x+c)$$

$$= x^3 + (a+b)x^2 + abx + cx^2 + c(a+b)x + abc$$

$$= x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$$

$$\boxed{(x+a)(x+b)(x+c) \equiv x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc}$$

1.2.3 Expansion of $(x \pm y)^3$

In the above identity by substituting $a = b = c = y$, we get

$$(x+y)(x+y)(x+y) = x^3 + (y+y+y)x^2 + [(y)(y)+(y)(y)+(y)(y)]x + (y)(y)(y)$$

$$(x+y)^3 = x^3 + (3y)x^2 + (3y^2)x + y^3$$
$$= x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x+y)^3 \equiv x^3 + 3x^2y + 3xy^2 + y^3$$
(or) $$(x+y)^3 \equiv x^3 + y^3 + 3xy(x+y)$$

Replacing y by $-y$ in the above identity, we get

$$(x-y)^3 \equiv x^3 - 3x^2y + 3xy^2 - y^3$$
(or) $$(x-y)^3 \equiv x^3 - y^3 - 3xy(x-y)$$

Using these identities of 1.2.2 and 1.2.3, let us solve the following problems.

Example 1.3

Find the product of

(i) $(x+2)(x+5)(x+7)$ (ii) $(a-3)(a-5)(a-7)$ (iii) $(2a-5)(2a+5)(2a-3)$

Solution

(i) $(x+2)(x+5)(x+7)$

$= x^3 + (2+5+7)x^2 + [(2)(5)+(5)(7)+(7)(2)]x + (2)(5)(7)$

$= x^3 + 14x^2 + (10+35+14)x + 70$

$= x^3 + 14x^2 + 59x + 70$

(ii) $(a-3)(a-5)(a-7) = [a+(-3)][a+(-5)][a+(-7)]$

$= a^3 + (-3-5-7)a^2 + [(-3)(-5)+(-5)(-7)+(-7)(-3)]a + (-3)(-5)(-7)$

$= a^3 - 15a^2 + (15+35+21)a - 105$

$= a^3 - 15a^2 + 71a - 105$

(iii) $(2a-5)(2a+5)(2a-3)=[2a+(-5)][2a+5][2a+(-3)]$

$= (2a)^3 + (-5+5-3)(2a)^2 + [(-5)(5)+(5)(-3)+(-3)(-5)](2a) + (-5)(5)(-3)$

$= 8a^3 + (-3)4a^2 + (-25-15+15)2a + 75$

$= 8a^3 - 12a^2 - 50a + 75$

Example 1.4

If $a+b+c=15$, $ab+bc+ca=25$ find $a^2+b^2+c^2$.

Solution We have $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$. So,

$$15^2 = a^2+b^2+c^2+2(25)$$

$$225 = a^2+b^2+c^2+50$$

$$\therefore a^2+b^2+c^2 = 225-50 = 175$$

Example 1.5

Expand (i) $(3a+4b)^3$ (ii) $(2x-3y)^3$

Solution

(i) $(3a+4b)^3 = (3a)^3 + 3(3a)^2(4b) + 3(3a)(4b)^2 + (4b)^3$

$= 27a^3 + 108a^2b + 144ab^2 + 64b^3$

(ii) $(2x-3y)^3 = (2x)^3 - 3(2x)^2(3y) + 3(2x)(3y)^2 - (3y)^3$

$= 8x^3 - 36x^2y + 54xy^2 - 27y^3$

Example 1.6

Evaluate each of the following using suitable identities.

(i) $(105)^3$ (ii) $(999)^3$

Solution

(i) $(105)^3 = (100+5)^3$

$=(100)^3 + (5)^3 + 3(100)(5)(100+5)$ $(\because (x+y)^3 = x^3+y^3+3xy(x+y))$

$= 1000000 + 125 + 1500(105)$

$= 1000000 + 125 + 157500 = 1157625$

(ii) $(999)^3 = (1000-1)^3$

$=(1000)^3 - (1)^3 - 3(1000)(1)(1000-1)$

$(\because (x-y)^3 = x^3 - y^3 - 3xy(x-y))$

$= 1000000000 - 1 - 3000(999)$

$= 1000000000 - 1 - 2997000 = 997002999$

Some Useful Identities involving sum ,difference and product of x and y

$$x^3+y^3 \equiv (x+y)^3-3xy(x+y)$$
$$x^3-y^3 \equiv (x-y)^3+3xy(x-y)$$

Let us solve some problems involving above identities.

Example 1.7

Find x^3+y^3 if $x+y=4$ and $xy=5$

Solution We know that $x^3+y^3 = (x+y)^3-3xy(x+y)$

$\therefore \quad x^3+y^3 = (4)^3-3(5)(4) = 64-60 = 4$

Example 1.8

Find x^3-y^3 if $x-y=5$ and $xy=16$

Solution We know that $x^3-y^3 = (x-y)^3+3xy(x-y)$

$\therefore \quad x^3-y^3 = (5)^3+3(16)(5) = 125+240 = 365$

Example 1.9

If $x+\frac{1}{x}=5$, find the value of $x^3+\frac{1}{x^3}$

Solution We know that $x^3+y^3 = (x+y)^3-3xy(x+y)$

Put $y=\frac{1}{x}$, $x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)$

$= (5)^3-3(5) = 125-15 = 110$

Example 1.10

If $y-\frac{1}{y}=9$, find the value of $y^3-\frac{1}{y^3}$

Solution We know that, $x^3-y^3 = (x-y)^3+3xy(x-y)$

Put $x=y$ *and* $y=\frac{1}{y}$, $y^3-\frac{1}{y^3} = \left(y-\frac{1}{y}\right)^3+3\left(y-\frac{1}{y}\right)$

$= (9)^3+3(9) = 729+27 = 756$

The following identity is frequently used in higher studies

$$x^3 + y^3 + z^3 - 3xyz \equiv (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

Note If $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$

Example 1.11

Simplify $(x + 2y + 3z)(x^2 + 4y^2 + 9z^2 - 2xy - 6yz - 3zx)$

Solution We know that, $(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = x^3 + y^3 + z^3 - 3xyz$

$\therefore$ $(x + 2y + 3z)(x^2 + 4y^2 + 9z^2 - 2xy - 6yz - 3zx)$

$= (x + 2y + 3z)[x^2 + (2y)^2 + (3z)^2 - (x)(2y) - (2y)(3z) - (3z)(x)]$

$= (x)^3 + (2y)^3 + (3z)^3 - 3(x)(2y)(3z)$

$= x^3 + 8y^3 + 27z^3 - 18xyz$

Example 1.12

Evaluate $12^3 + 13^3 - 25^3$

Solution Let $x = 12,\ y = 13,\ z = -25$. Then

$$x + y + z = 12 + 13 - 25 = 0$$

If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

$$\therefore 12^3 + 13^3 - 25^3 = 12^3 + 13^3 + (-25)^3 = 3(12)(13)(-25) = -11700$$

Exercise 1.1

1. Expand the following

   (i) $(5x + 2y + 3z)^2$ (ii) $(2a + 3b - c)^2$ (iii) $(x - 2y - 4z)^2$ (iv) $(p - 2q + r)^2$

2. Find the expansion of

   (i) $(x + 1)(x + 4)(x + 7)$ (ii) $(p + 2)(p - 4)(p + 6)$

   (iii) $(x + 5)(x - 3)(x - 1)$ (iv) $(x - a)(x - 2a)(x - 4a)$

   (v) $(3x + 1)(3x + 2)(3x + 5)$ (vi) $(2x + 3)(2x - 5)(2x - 7)$

3. Using algebraic identities find the coefficients of x^2 term, x term and constant term.

   (i) $(x + 7)(x + 3)(x + 9)$ (ii) $(x - 5)(x - 4)(x + 2)$

   (iii) $(2x + 3)(2x + 5)(2x + 7)$ (iv) $(5x + 2)(1 - 5x)(5x + 3)$

4. If $(x+a)(x+b)(x+c) \equiv x^3 - 10x^2 + 45x - 15$ find $a+b+c$, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ and $a^2+b^2+c^2$.

5. Expand : (i) $(3a+5b)^3$ (ii) $(4x-3y)^3$ (iii) $\left(2y-\frac{3}{y}\right)^3$

6. Evaluate : (i) 99^3 (ii) 101^3 (iii) 98^3 (iv) 102^3 (v) 1002^3

7. Find $8x^3+27y^3$ if $2x+3y=13$ and $xy=6$.

8. If $x-y=-6$ and $xy=4$, find the value of x^3-y^3.

9. If $x+\frac{1}{x}=4$, find the value of $x^3+\frac{1}{x^3}$.

10. If $x-\frac{1}{x}=3$, find the value of $x^3-\frac{1}{x^3}$.

11. Simplify : (i) $(2x+y+4z)(4x^2+y^2+16z^2-2xy-4yz-8zx)$

(ii) $(x-3y-5z)(x^2+9y^2+25z^2+3xy-15yz+5zx)$

12. Evaluate using identities : (i) $6^3-9^3+3^3$ (ii) $16^3-6^3-10^3$

1.3 Factorization of Polynomials

We have seen how the distributive property may be used to expand a product of algebraic expressions into sum or difference of expressions.

For example,

(i) $x(x+y)=x^2+xy$ (ii) $x(y-z)=xy-xz$

(iii) $a(a^2-2a+1)=a^3-2a^2+a$

Now, we will learn how to convert a sum or difference of expressions into a product of expressions.

Now, consider $ab+ac$. Using the distributive law, $a(b+c)=ab+ac$, by writing in the reverse direction $ab+ac$ is $a(b+c)$. This process of expressing $ab+ac$ into $a(b+c)$ is known as factorization. In both the terms, ab and ac 'a' is the common factor. Similarly,

$5m+15=5(m)+5(3)=5(m+3)$.

In $b(b-5)+g(b-5)$ clearly $(b-5)$ is a common factor.

$b(b-5)+g(b-5)=(b-5)(b+g)$

Example 1.13

Factorize the following

(i) $pq + pr - 3ps$ (ii) $4a - 8b + 5ax - 10bx$ (iii) $2a^3 + 4a^2$ (iv) $6a^5 - 18a^3 + 42a^2$

Solution

(i) $pq + pr - 3ps = p(q + r - 3s)$

(ii) $4a - 8b + 5ax - 10bx = (4a - 8b) + (5ax - 10bx)$

$= 4(a - 2b) + 5x(a - 2b) = (a - 2b)(4 + 5x)$

(iii) $2a^3 + 4a^2$

Highest common factor is $2a^2$

$\therefore 2a^3 + 4a^2 = 2a^2(a + 2)$.

(iv) $6a^5 - 18a^3 + 42a^2$

Highest common factor is $6a^2$

$\therefore 6a^5 - 18a^3 + 42a^2 = 6a^2(a^3 - 3a + 7)$

1.3.1 Factorization Using Identities

(i) $a^2 + 2ab + b^2 \equiv (a + b)^2$

(ii) $a^2 - 2ab + b^2 \equiv (a - b)^2$ (or) $a^2 - 2ab + b^2 \equiv (-a + b)^2$

(iii) $a^2 - b^2 \equiv (a + b)(a - b)$

(iv) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \equiv (a + b + c)^2$

Example 1.14

Factorize (i) $4x^2 + 12xy + 9y^2$ (ii) $16a^2 - 8a + 1$ (iii) $9a^2 - 16b^2$

(iv) $(a + b)^2 - (a - b)^2$ (v) $25(a + 2b - 3c)^2 - 9(2a - b - c)^2$ (vi) $x^5 - x$

Solution

(i) $4x^2 + 12xy + 9y^2 = (2x)^2 + 2(2x)(3y) + (3y)^2 = (2x + 3y)^2$

(ii) $16a^2 - 8a + 1 = (4a)^2 - 2(4a)(1) + (1)^2 = (4a - 1)^2$ or $(1 - 4a)^2$

(iii) $9a^2 - 16b^2 = (3a)^2 - (4b)^2 = (3a + 4b)(3a - 4b)$

(iv) $(a+b)^2-(a-b)^2 = [(a+b)+(a-b)][(a+b)-(a-b)]$

$= (a+b+a-b)(a+b-a+b) = (2a)(2b) = (4)(a)(b)$

(v) $25(a+2b-3c)^2 - 9(2a-b-c)^2 = [5(a+2b-3c)]^2 - [3(2a-b-c)]^2$

$= [5(a+2b-3c)+3(2a-b-c)][5(a+2b-3c)-3(2a-b-c)]$

$= (5a+10b-15c+6a-3b-3c)(5a+10b-15c-6a+3b+3c)$

$= (11a+7b-18c)(-a+13b-12c)$

(vi) $x^5 - x = x(x^4-1) = x[(x^2)^2-(1)^2]$

$= x(x^2+1)(x^2-1) = x(x^2+1)[(x)^2-(1)^2]$

$= x(x^2+1)(x+1)(x-1)$

1.3.2 Factorization Using the Identity

$a^2+b^2+c^2+2ab+2bc+2ca \equiv (a+b+c)^2$

Example 1.15

Factorize $a^2+4b^2+36-4ab-24b+12a$

Solution $a^2+4b^2+36-4ab-24b+12a$ can be written as

$(a)^2+(-2b)^2+(6)^2+2(a)(-2b)+2(-2b)(6)+2(6)(a) = (a-2b+6)^2$ or

$(-a)^2+(2b)^2+(-6)^2+2(-a)(2b)+2(2b)(-6)+2(-6)(-a) = (-a+2b-6)^2$

That is $(a-2b+6)^2 = [(-1)(-a+2b-6)^2] = (-1)^2(-a+2b-6)^2 = (-a+2b-6)^2$

Example 1.16

Factorize $4x^2+y^2+9z^2-4xy+6yz-12zx$

Solution $4x^2+y^2+9z^2-4xy+6yz-12zx$

$= (2x)^2+(-y)^2+(-3z)^2+2(2x)(-y)+2(-y)(-3z)+2(-3z)(2x)$

$= (2x-y-3z)^2$ or $(-2x+y+3z)^2$

1.3.3 Factorization of $x^3 + y^3$ and $x^3 - y^3$

We have $x^3 + 3x^2y + 3xy^2 + y^3 = (x+y)^3$. So,

$$x^3 + y^3 + 3xy(x+y) = (x+y)^3$$

$$\begin{aligned} \Longrightarrow x^3 + y^3 &= (x+y)^3 - 3xy(x+y) \\ &= (x+y)[(x+y)^2 - 3xy] \\ &= (x+y)(x^2 + 2xy + y^2 - 3xy) \\ &= (x+y)(x^2 - xy + y^2) \end{aligned}$$

$$\boxed{x^3 + y^3 \equiv (x+y)(x^2 - xy + y^2)}$$

We have $x^3 - 3x^2y + 3xy^2 - y^3 = (x-y)^3$. So,

$$x^3 - y^3 - 3xy(x-y) = (x-y)^3$$

$$\begin{aligned} \Longrightarrow x^3 - y^3 &= (x-y)^3 + 3xy(x-y) \\ &= (x-y)[(x-y)^2 + 3xy] \\ &= (x-y)(x^2 - 2xy + y^2 + 3xy) \\ &= (x-y)(x^2 + xy + y^2) \end{aligned}$$

$$\boxed{x^3 - y^3 \equiv (x-y)(x^2 + xy + y^2)}$$

Using the above identities let us factorize the following expressions.

Example 1.17

Factorize (i) $8x^3 + 125y^3$ (ii) $27x^3 - 64y^3$

Solution

(i)
$$\begin{aligned} 8x^3 + 125y^3 &= (2x)^3 + (5y)^3 \\ &= (2x+5y)[(2x)^2 - (2x)(5y) + (5y)^2] \\ &= (2x+5y)(4x^2 - 10xy + 25y^2) \end{aligned}$$

(ii)
$$\begin{aligned} 27x^3 - 64y^3 &= (3x)^3 - (4y)^3 \\ &= (3x-4y)[(3x)^2 + (3x)(4y) + (4y)^2] \\ &= (3x-4y)(9x^2 + 12xy + 16y^2) \end{aligned}$$

Exercise 1.2

1. Factorize the following expressions:

(i) $2a^3 - 3a^2b + 2a^2c$ (ii) $16x + 64x^2y$ (iii) $10x^3 - 25x^4y$

(iv) $xy - xz + ay - az$ (v) $p^2 + pq + pr + qr$

2. Factorize the following expressions:

(i) $x^2 + 2x + 1$ (ii) $9x^2 - 24xy + 16y^2$

(iii) $b^2 - 4$ (iv) $1 - 36x^2$

3. Factorize the following expressions:

(i) $p^2 + q^2 + r^2 + 2pq + 2qr + 2rp$ (ii) $a^2 + 4b^2 + 36 - 4ab + 24b - 12a$

(iii) $9x^2 + y^2 + 1 - 6xy + 6x - 2y$ (iv) $4a^2 + b^2 + 9c^2 - 4ab - 6bc + 12ca$

(v) $25x^2 + 4y^2 + 9z^2 - 20xy + 12yz - 30zx$

4. Factorize the following expressions:

(i) $27x^3 + 64y^3$ (ii) $m^3 + 8$ (iii) $a^3 + 125$

(iv) $8x^3 - 27y^3$ (v) $x^3 - 8y^3$

1.3.4 Factorization of the Quadratic Polynomials of the type $ax^2 + bx + c$; $a \neq 0$

So far we have used the identities to factorize certain types of polynomials. In this section we will learn, without identities how to resolve quadratic polynomials into two linear polynomials when (i) $a = 1$ and (ii) $a \neq 1$

(i) Factorizing the quadratic polynomials of the type $x^2 + bx + c$.

suppose $(x + p)$ and $(x + q)$ are the two factors of $x^2 + bx + c$. Then we have

$$x^2 + bx + c = (x + p)(x + q)$$

$$= x(x + p) + q(x + p)$$

$$= x^2 + px + qx + pq$$

$$= x^2 + (p + q)x + pq$$

This implies that the two numbers p and q are chosen in such way that $c = pq$ and $b = p + q$ to get $x^2 + bx + c = (x + p)(x + q)$

We use this basic idea to factorize the following problems

For example,

(1) $x^2 + 8x + 15 = (x+3)(x+5)$

here $c = 15 = 3 \times 5$ and $3 + 5 = 8 = b$

(2) $x^2 - 5x + 6 = (x-2)(x-3)$

here $c = 6 = (-2)\times(-3)$ and $(-2)+(-3) = -5 = b$

(3) $x^2 + x - 2 = (x+2)(x-1)$

here $c = -2 = (+2)\times(-1)$ and $(+2)+(-1) = 1 = b$

(4) $x^2 - 4x - 12 = (x-6)(x+2)$

here $c = -12 = (-6)\times(+2)$ and $(-6)+(+2) = -4 = b$

In the above examples the constant term is split into two factors such that their sum is equal to the coefficients of x.

Example 1.18

Factorize the following.

(i) $x^2 + 9x + 14$ (ii) $x^2 - 9x + 14$ (iii) $x^2 + 2x - 15$ (iv) $x^2 - 2x - 15$

Solution

(i) $x^2 + 9x + 14$

To factorize we have to find p and q, such that $pq = 14$ and $p + q = 9$.

$$\begin{aligned} x^2 + 9x + 14 &= x^2 + 2x + 7x + 14 \\ &= x(x+2) + 7(x+2) \\ &= (x+2)(x+7) \\ \therefore\ x^2 + 9x + 14 &= (x+7)(x+2) \end{aligned}$$

Factors of 14	Sum of factors
1, 14	15
2, 7	9
The required factors are 2, 7	

(ii) $x^2 - 9x + 14$

To factorize we have to find p and q such that $pq = 14$ and $p + q = -9$

$$x^2 - 9x + 14 = x^2 - 2x - 7x + 14$$
$$= x(x-2) - 7(x-2)$$
$$= (x-2)(x-7)$$
$$\therefore x^2 - 9x + 14 = (x-2)(x-7)$$

Factors of 14	Sum of factors
–1, –14	–15
–2, –7	–9
The required factors are –2, –7	

(iii) $x^2 + 2x - 15$

To factorize we have to find p and q, such that $pq = -15$ and $p + q = 2$

$$x^2 + 2x - 15 = x^2 - 3x + 5x - 15$$
$$= x(x-3) + 5(x-3)$$
$$= (x-3)(x+5)$$
$$\therefore x^2 + 2x - 15 = (x-3)(x+5)$$

Factors of –15	Sum of factors
–1, 15	14
–3, 5	2
The required factors are –3, 5	

(iv) $x^2 - 2x - 15$

To factorize we have to find p and q, such that $pq = -15$ and $p + q = -2$

$$x^2 - 2x - 15 = x^2 + 3x - 5x - 15$$
$$= x(x+3) - 5(x+3)$$
$$= (x+3)(x-5)$$
$$\therefore x^2 - 2x - 15 = (x+3)(x-5)$$

Factors of –15	Sum of factors
1, –15	–14
3, –5	–2
The required factors are 3, –5	

(ii) Factorizing the quadratic polynomials of the type ax^2+bx+c.

Since a is different from 1, the linear factors of ax^2+bx+c will be of the form $(rx+p)$ and $(sx+q)$.

Then, $ax^2+bx+c = (rx+p)(sx+q)$

$= rsx^2+(ps+qr)x+pq$

Comparing the coefficients of x^2, we get $a=rs$. Similarly, comparing the coefficients of x, we get $b=ps+qr$. And, on comparing the constant terms, we get $c=pq$.

This shows us that b is the sum of two numbers ps and qr, whose product is $(ps)\times(qr)=(pr)\times(sq)=ac$

Therefore, to factorize ax^2+bx+c, we have to write b as the sum of two numbers whose product is ac.

The following steps to be followed to factorize ax^2+bx+c

Step1 : Multiply the coefficient of x^2 and constant term.

Step2 : Split this product into two factors such that their sum is equal to the coefficient of x.

Step3 : The terms are grouped into two pairs and factorize.

Example 1.19

Factorize the following

(i) $2x^2+15x+27$ (ii) $2x^2-15x+27$

(iii) $2x^2+15x-27$ (iv) $2x^2-15x-27$

Solution

(i) $2x^2+15x+27$

Coefficient of $x^2 = 2$; constant term $= 27$

Their product $= 2\times 27 = 54$

Coefficient of $x = 15$

$\therefore$ product $= 54$; sum $= 15$

Factors of 54	Sum of factors
1, 54	55
2, 27	29
3, 18	21
6, 9	15
The required factors are 6, 9	

$2x^2+15x+27 = 2x^2+6x+9x+27$

$= 2x(x+3)+9(x+3)$

$= (x+3)(2x+9)$

$\therefore\ 2x^2+15x+27 = (x+3)(2x+9)$

(ii) $2x^2 - 15x + 27$

Coefficient of $x^2 = 2$; constant term = 27

Their product= $2 \times 27 = 54$

Coefficient of $x = -15$

$\therefore$ product= 54 ; sum = – 15

Factors of 54	Sum of factors
–1, –54	–55
–2, –27	–29
–3, –18	–21
–6, –9	–15
The required factors are –6, –9	

$$2x^2 - 15x + 27 = 2x^2 - 6x - 9x + 27$$
$$= 2x(x-3) - 9(x-3)$$
$$= (x-3)(2x-9)$$
$$\therefore 2x^2 - 15x + 27 = (x-3)(2x-9)$$

(iii) $2x^2 + 15x - 27$

Coefficient of $x^2 = 2$; constant term = – 27

Their product $= 2 \times -27 = -54$

Coefficient of $x = 15$

$\therefore$ product = –54; sum = 15

Factors of –54	Sum of factors
–1, 54	53
–2, 27	25
–3, 18	15
The required factors are –3, 18	

$$2x^2 + 15x - 27 = 2x^2 - 3x + 18x - 27$$
$$= x(2x-3) + 9(2x-3)$$
$$= (2x-3)(x+9)$$
$$\therefore 2x^2 + 15x - 27 = (2x-3)(x+9)$$

(iv) $2x^2 - 15x - 27$

Coefficient of $x^2 = 2$; constant term = – 27

Their product $= 2 \times -27 = -54$

Coefficient of $x = -15$

$\therefore$ product = –54; sum = –15

Factors of –54	Sum of factors
1, –54	–53
2, –27	–25
3, –18	–15
The required factors are 3, –18	

$$2x^2 - 15x - 27 = 2x^2 + 3x - 18x - 27$$
$$= x(2x+3) - 9(2x+3)$$
$$= (2x+3)(x-9)$$
$$\therefore 2x^2 - 15x - 27 = (2x+3)(x-9)$$

Example 1.20

Factorize $(x+y)^2+9(x+y)+8$

Solution Let $x+y=p$

Then the equation is p^2+9p+8

Coefficient of $p^2=1$; constant term $=8$

Their product$=1\times 8=8$

Coefficient of $p=9$

Factors of 8	Sum of factors
1, 8	9
The required factors are 1, 8	

$\therefore$ product $=8$; sum $=9$

$$p^2+9p+8=p^2+p+8p+8$$

$$=p(p+1)+8(p+1)$$

$$=(p+1)(p+8)$$

substituting, $p=x+y$

$$\therefore\ (x+y)^2+9(x+y)+8=(x+y+1)(x+y+8)$$

Example 1.21

Factorize : (i) x^3-2x^2-x+2 (ii) x^3+3x^2-x-3

Solution

(i) Let $p(x)=x^3-2x^2-x+2$

$p(x)$ is a cubic polynomial, so it may have three linear factors.

The constant term is 2. The factors of 2 are -1, 1, -2 and 2.

$p(-1)=(-1)^3-2(-1)^2-(-1)+2=-1-2+1+2=0$

$\therefore$ $(x+1)$ is a factor of $p(x)$.

$p(1)=(1)^3-2(1)^2-1+2=1-2-1+2=0$

$\therefore$ $(x-1)$ is a factor of $p(x)$.

$p(-2)=(-2)^3-2(-2)^2-(-2)+2=-8-8+2+2=-12\neq 0$

$\therefore$ $(x+2)$ is not a factor of $p(x)$.

$p(2)=(2)^3-2(2)^2-2+2=8-8-2+2=0$

$\therefore$ $(x-2)$ is a factor of $p(x)$.

The three factors of $p(x)$ are $(x+1),(x-1)$ and $(x-2)$

$\therefore$ $x^3-2x^2-x+2=(x+1)(x-1)(x-2)$.

Another method

$$x^3-2x^2-x+2=x^2(x-2)-1(x-2)$$

$= (x-2)(x^2-1)$

$= (x-2)(x+1)(x-1) \qquad [(\because a^2-b^2=(a+b)(a-b)]$

(ii) Let $p(x) = x^3+3x^2-x-3$

$p(x)$ is a cubic polynomial, so it may have three linear factors.

The constant term is –3. The factors of –3 are –1, 1,–3 and 3.

$p(-1) = (-1)^3+3(-1)^2-(-1)-3 = -1+3+1-3 = 0$

$\therefore (x+1)$ is a factor of $p(x)$.

$p(1) = (1)^3+3(1)^2-1-3 = 1+3-1-3 = 0$

$\therefore (x-1)$ is a factor of $p(x)$.

$p(-3) = (-3)^3+3(-3)^2-(-3)-3 = -27+27+3-3 = 0$

$\therefore (x+3)$ is a factor of $p(x)$.

The three factors of $p(x)$ are $(x+1), (x-1)$ and $(x+3)$

$\therefore x^3+3x^2-x-3 = (x+1)(x-1)(x+3)$.

Exercise 1.3

1. Factorize each of the following.

(i) $x^2+15x+14$ (ii) $x^2+13x+30$ (iii) $y^2+7y+12$

(iv) $x^2-14x+24$ (v) $y^2-16y+60$ (vi) $t^2-17t+72$

(vii) $x^2+14x-15$ (viii) $x^2+9x-22$ (ix) $y^2+5y-36$

(x) $x^2-2x-99$ (xi) $m^2-10m-144$ (xii) y^2-y-20

2. Factorize each of the following.

(i) $3x^2+19x+6$ (ii) $5x^2+22x+8$ (iii) $2x^2+9x+10$

(iv) $14x^2+31x+6$ (v) $5y^2-29y+20$ (vi) $9y^2-16y+7$

(vii) $6x^2-5x+1$ (viii) $3x^2-10x+8$ (ix) $3x^2+5x-2$

(x) $2a^2+17a-30$ (xi) $11+5x-6x^2$ (xii) $8x^2+29x-12$

(xiii) $2x^2-3x-14$ (xiv) $18x^2-x-4$ (xv) $10-7x-3x^2$

3. Factorize the following

(i) $(a+b)^2+9(a+b)+14$ (ii) $(p-q)^2-7(p-q)-18$

4. Factorize the following

(i) x^3+2x^2-x-2 (ii) x^3-3x^2-x+3

(iii) x^3+x^2-4x-4 (iv) x^3+5x^2-x-5

Example 1.22

Solve the following pair of equations by substitution method.

$2x + 5y = 2$ and $x + 2y = 3$

Solution We have $2x + 5y = 2$ (1)

$x + 2y = 3$ (2)

Equation (2) becomes, $x = 3 - 2y$ (3)

Substituting x in (1) we get, $2(3 - 2y) + 5y = 2$

$$\Longrightarrow 6 - 4y + 5y = 2$$

$$-4y + 5y = 2 - 6$$

$$\therefore y = -4$$

Substituting $y = -4$ in (3), we get, $x = 3 - 2(-4) = 3 + 8 = 11$

$\therefore$ The solution is $x = 11$ and $y = -4$

Example 1.23

Solve $x + 3y = 16$, $2x - y = 4$ by using substitution method.

Solution

We have $x + 3y = 16$ (1)

$2x - y = 4$ (2)

Equation (1) becomes, $x = 16 - 3y$ (3)

Substituting x in (2) we get, $2(16 - 3y) - y = 4$

$$\Longrightarrow 32 - 6y - y = 4$$

$$-6y - y = 4 - 32$$

$$-7y = -28$$

$$y = \frac{-28}{-7} = 4$$

Substituting $y = 4$ in (3) we get, $x = 16 - 3(4)$

$$= 16 - 12 = 4$$

$\therefore$ The solution is $x = 4$ and $y = 4$.

Example 1.24

Solve by substitution method $\frac{1}{x}+\frac{1}{y}=4$ and $\frac{2}{x}+\frac{3}{y}=7, \quad x \neq 0, y \neq 0$

Solution

Let $\frac{1}{x}=a$ and $\frac{1}{y}=b$

The given equations become

$$a+b=4 \qquad (1)$$

$$2a+3b=7 \qquad (2)$$

Equation (1) becomes $b=4-a \qquad (3)$

Substituting b in (2) we get, $2a+3(4-a)=7$

$$\Longrightarrow 2a+12-3a=7$$

$$2a-3a=7-12$$

$$-a=-5 \Longrightarrow a=5$$

Substituting $a=5$ in (3) we get, $b=4-5=-1$

But $\frac{1}{x}=a \Rightarrow x=\frac{1}{a}=\frac{1}{5}$

$\frac{1}{y}=b \Rightarrow y=\frac{1}{b}=\frac{1}{-1}=-1$

$\therefore$ The solution is $x=\frac{1}{5}, y=-1$

Example 1.25

The cost of a pen and a note book is ₹ 60. The cost of a pen is ₹ 10 less than that of a notebook. Find the cost of each.

Solution

Let the cost of a pen = ₹ x

Let the cost of a note book = ₹ y

From given data we have

$x + y = 60$ (1)

$x = y - 10$ (2)

Substituting x in (1) we get, $y - 10 + y = 60$

$\Longrightarrow y + y = 60 + 10 \Longrightarrow 2y = 70$

$\therefore y = \dfrac{70}{2} = 35$

Substituting $y = 35$ in (2) we get, $x = 35 - 10 = 25$

$\therefore$ The cost of a pen is ₹ 25.

The cost of a note book is ₹ 35.

Example 1.26

The cost of three mathematics books and four science books is ₹ 216. The cost of three mathematics books is the same as that of four science books. Find the cost of each book.

Solution

Let the cost of a mathematics book be ₹ x and cost of a science book be ₹ y.

By given data,

$3x + 4y = 216$ (1)

$3x = 4y$ (2)

The equation (2) becomes, $x = \dfrac{4y}{3}$ (3)

Substituting x in (1) we get, $3\left(\dfrac{4y}{3}\right) + 4y = 216$

$\Longrightarrow 4y + 4y = 216 \Longrightarrow 8y = 216$

$$\therefore\ y = \frac{216}{8} = 27$$

substituting y =27 in (3) we get, $x = \frac{4(27)}{3} = 36$

$\therefore$ The cost of one mathematics book = ₹ 36.

The cost of one science book = ₹ 27.

Example 1.27

From Dharmapuri bus stand if we buy 2 tickets to Palacode and 3 tickets to Karimangalam the total cost is ₹ 32, but if we buy 3 tickets to Palacode and one ticket to Karimangalam the total cost is ₹ 27. Find the fares from Dharmapuri to Palacode and to Karimangalam.

Solution

Let the fare from Dharmapuri to Palacode be ₹ x and to Karimangalam be ₹ y.

From the given data, we have

$2x + 3y = 32$ (1)

$3x + y = 27$ (2)

Equation (2) becomes, $y = 27 - 3x$ (3)

Substituting y in (1) we get, $2x + 3(27 - 3x) = 32$

$$\Longrightarrow 2x + 81 - 9x = 32$$

$$2x - 9x = 32 - 81$$

$$-7x = -49$$

$$-7x = -49$$

$$\therefore x = \frac{-49}{-7} = 7$$

Substituting $x = 7$ in (3) we get, $y = 27 - 3(7) = 27 - 21 = 6$

$\therefore$ The fare from Dharmapuri to Palacode is ₹ 7 and to Karimangalam is ₹ 6.

Example 1.28

The sum of two numbers is 55 and their difference is 7. Find the numbers .

Solution

Let the two numbers be x and y, where $x > y$

By the given data, $x + y = 55$ (1)

$x - y = 7$ (2)

Equation (2) becomes, $x = 7 + y$ (3)

Substituting x in (1) we get, $7 + y + y = 55$

$$\Longrightarrow 2y = 55 - 7 = 48$$

$$\therefore y = \frac{48}{2} = 24$$

Substituting $y = 24$ in (3) we get, $x = 7 + 24 = 31$.

$\therefore$ The required two numbers are 31 and 24.

Example 1.29

A number consist of two digits whose sum is 11. The number formed by reversing the digits is 9 less than the original number. Find the number.

Solution

Let the tens digit be x and the units digit be y. Then the number is $10x + y$.

Sum of the digits is $x + y = 11$ (1)

The number formed by reversing the digits is $10y + x$.

Given data, $(10x + y) - 9 = 10y + x$

$$\Longrightarrow 10x + y - 10y - x = 9$$

$$9x - 9y = 9$$

Dividing by 9 on both sides, $x - y = 1$ (2)

Equation (2) becomes $x = 1 + y$ (3)

Substituting x in (1) we get, $1 + y + y = 11$

$$\Longrightarrow 2y + 1 = 11$$

$$2y = 11 - 1 = 10$$

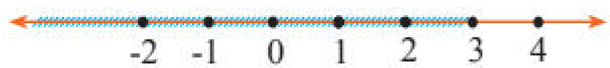
$$\therefore y = \frac{10}{2} = 5$$

Substituting $y = 5$ in (3) we get, $x = 1 + 5 = 6$

$\therefore$ The number is $10x + y = 10(6) + 5 = 65$

Example 1.30

Solve $4(x-1) \le 8$



Solution

$4(x-1) \le 8$

Dividing by 4 on both sides,

$x - 1 \le 2$

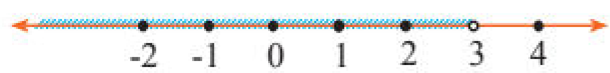
$\Longrightarrow \quad x \le 2 + 1 \Longrightarrow x \le 3$

The real numbers less than or equal to 3 are solutions of given inequation.

Shaded circle indicates that point is included in the solution set.

Example 1.31

Solve $3(5-x) > 6$



Solution We have, $3(5-x) > 6$

Dividing by 3 on both sides, $5 - x > 2$

$\Longrightarrow -x > 2 - 5 \Longrightarrow -x > -3$

$\therefore x < 3$ (See remark given below)

The real numbers less than 3 are solutions of given inequation.

Remark (i) $-a > -b \Longrightarrow a < b$ (ii) $a < b \Longrightarrow \frac{1}{a} > \frac{1}{b}$ where $a \neq 0,\ b \neq 0$

(iii) $a < b \Longrightarrow ka < kb$ for $k > 0$ (iv) $a < b \Longrightarrow ka > kb$ for $k < 0$

Example 1.32

Solve $3 - 5x \leq 9$

Solution We have, $3 - 5x \leq 9$

$\Longrightarrow -5x \leq 9 - 3 \Longrightarrow -5x \leq 6$

$\Longrightarrow 5x \geq -6 \Longrightarrow x \geq -\frac{6}{5} \Longrightarrow x \geq -1.2$

The real numbers greater than or equal to -1.2 are solutions of given inequation.

Exercise 1.4

1. Solve the following equations by substitution method .

(i) $x + 3y = 10;\ 2x + y = 5$ (ii) $2x + y = 1;\ 3x - 4y = 18$

(iii) $5x + 3y = 21;\ 2x - y = 4$ (iv) $\frac{1}{x} + \frac{2}{y} = 9;\ \frac{2}{x} + \frac{1}{y} = 12 \quad (x \neq 0, y \neq 0)$

(v) $\frac{3}{x} + \frac{1}{y} = 7;\ \frac{5}{x} - \frac{4}{y} = 6\ (x \neq 0, y \neq 0)$

2. Find two numbers whose sum is 24 and difference is 8.

3. A number consists of two digits whose sum is 9. The number formed by reversing the digits exceeds twice the original number by 18. Find the original number.

4 Kavi and Kural each had a number of apples . Kavi said to Kural "If you give me 4 of your apples, my number will be thrice yours". Kural replied "If you give me 26, my number will be twice yours". How many did each have with them?.

5. Solve the following inequations.

(i) $2x + 7 > 15$ (ii) $2(x - 2) < 3$ (iii) $2(x + 7) \leq 9$ (iv) $3x + 14 \geq 8$

- $(x + y + z)^2 \equiv x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
- $(x + y)^3 \equiv x^3 + y^3 + 3xy(x + y)$ $\qquad x^3 + y^3 \equiv (x + y)(x^2 - xy + y^2)$
- $(x - y)^3 \equiv x^3 - y^3 - 3xy(x - y)$ $\qquad x^3 - y^3 \equiv (x - y)(x^2 + xy + y^2)$
- $x^3 + y^3 + z^3 - 3xyz \equiv (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
- $(x + a)(x + b)(x + c) \equiv x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

2.2.2 Pythagoras Theorem

The Pythagoras theorem is a tool to solve for unknown values on right triangle.

Pythagoras Theorem: The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

This relationship is useful in solving many problems and in developing trigonometric concepts.

2.2.3 Trigonometric Ratios

Consider the right triangle in the Fig. 2.2. In the right triangle, we refer to the lengths of the three sides according to how they are placed in relation to the angle θ

- The side that is opposite to the right angle is called the *Hypotenuse*. This is the longest side in a right triangle.
- The side that is opposite to the angle θ is called the *Opposite side*.
- The side that runs alongside the angle θ and which is not the Hypotenuse is called the *Adjacent side*.

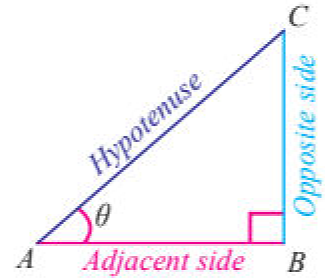


Fig. 2.2

3.3 Mean

3.3.1 Arithmetic Mean - Raw Data

The arithmetic mean is the sum of a set of observations, positive, negative or zero, divided by the number of observations. If we have n real numbers $x_1, x_2, x_3 ...., x_n$, then their arithmetic mean, denoted by $\overline{x}$, is given by

$$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} \text{ or } \overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \text{ or } \overline{x} = \frac{\sum x}{n}$$

Remark $\overline{x} = \frac{\sum x}{n} \Longrightarrow n\overline{x} = \sum x$. That is,

Total number of observations × Mean = Sum of all observations

Example 3.4

Find the arithmetic mean of the marks 72, 73, 75, 82, 74 obtained by a student in 5 subjects in an annual examination.

Solution Here n = 5

$$\overline{x} = \frac{\sum x}{n} = \frac{72 + 73 + 75 + 82 + 74}{5} = \frac{376}{5} = 75.2$$

$\therefore$ Mean = 75.2

Example 3.5

The mean of the 5 numbers is 32. If one of the numbers is excluded, then the mean is reduced by 4. Find the excluded number.

Solution

Mean of 5 numbers = 32.

Sum of these numbers = $32 \times 5 = 160$ $\quad (\because n\overline{x} = \sum x)$

Mean of 4 numbers = $32 - 4 = 28$

Sum of these 4 numbers = $28 \times 4 = 112$

Excluded number = (Sum of the 5 given numbers) − (Sum of the 4 numbers)

= $160 - 112 = 48$

Example 3.6

Obtain the mean of the following data.

x	5	10	15	20	25
f	3	10	25	7	5

Solution

x	f	fx
5	3	15
10	10	100
15	25	375
20	7	140
25	5	125
Total	$\sum f = 50$	$\sum fx = 755$

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{755}{50} = 15.1$$

Mean = 15.1

3.4 Median

Median is defined as the middle item of the given observations arranged in order.

3.4.1 Median - Raw Data

Steps:

(i) Arrange the n given numbers in ascending or descending order of magnitude.

(ii) When n is odd, $\left(\frac{n+1}{2}\right)^{th}$ observation is the median.

(iii) When n is even the median is the arithmetic mean of the two middle values.

i.e., when n is even,

Median = Mean of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations.

Example 3.10

Find the median of the following numbers

(i) 24, 22, 23, 14, 15, 7, 21 (ii) 17, 15, 9, 13, 21, 32, 42, 7, 12, 10.

Solution

(i) Let us arrange the numbers in ascending order as below.

7, 14, 15, 21, 22, 23, 24

Number of items $n = 7$

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ observation} \quad (\because n \text{ is odd})$$

$$= \left(\frac{7+1}{2}\right)^{th} \text{ observation}$$

$$= 4^{th} \text{ observation} \quad = 21$$

(ii) Let us arrange the numbers in ascending order

7, 9, 10, 12, 13, 15, 17, 21, 32, 42.

Number of items $n = 10$

Median is the mean of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations. $(\because n$ is even)

$$\left(\frac{n}{2}\right)^{th} \text{ observation} = \left(\frac{10}{2}\right)^{th} \text{ observation} = 5^{th} \text{ observation} = 13$$

$$\left(\frac{n}{2}+1\right)^{th} \text{ observation} = 6^{th} \text{ observation} = 15.$$

$$\therefore \text{Median} = \frac{13+15}{2} = 14$$

Exercise 3.5

Choose the Correct Answer

1. The mean of the first 10 natural numbers is
   (A) 25 (B) 55 (C) 5.5 (D) 2.5
2. The Arithmetic mean of integers from –5 to 5 is
   (A) 3 (B) 0 (C) 25 (D) 10
3. If the mean of x, $x+2$, $x+4$, $x+6$, $x+8$ is 20 then x is
   (A) 32 (B) 16 (C) 8 (D) 4
4. The mode of the data 5, 5, 5, 5, 5, 1, 2, 2, 3, 3, 3, 4, 4, 4, 4 is
   (A) 2 (B) 3 (C) 4 (D) 5
5. The median of 14, 12, 10, 9, 11 is
   (A) 11 (B) 10 (C) 9.5 (D) 10.5
6. The median of 2, 7, 4, 8, 9, 1 is
   (A) 4 (B) 6 (C) 5.5 (D) 7
7. The mean of first 5 whole number is
   (A) 2 (B) 2.5 (C) 3 (D) 0
8. The Arithmetic mean of 10 number is –7. If 5 is added to every number, then the new Arithmetic mean is
   (A) –2 (B) 12 (C) –7 (D) 17
9. The Arithmetic mean of all the factors of 24 is
   (A) 8.5 (B) 5.67 (C) 7 (D) 7.5
10. The mean of 5 numbers is 20. If one number is excluded their mean is 15. Then the excluded number is
   (A) 5 (B) 40 (C) 20 (D) 10.

1.2 Surds

We know that $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are irrational numbers. These are square roots of rational numbers, which cannot be expressed as squares of any rational number. $\sqrt[3]{2}$, $\sqrt[3]{3}$, $\sqrt[3]{7}$ etc. are the cube roots of rational numbers, which cannot be expressed as cubes of any rational number. This type of irrational numbers are called surds or radicals.

Key Concept **Surds**

If 'a' is a positive rational number and n is a positive integer such that $\sqrt[n]{a}$ is an irrational number, then $\sqrt[n]{a}$ is called a 'surd' or a 'radical'.

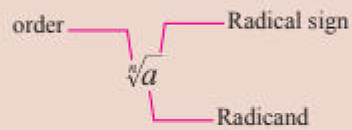
Notation

The general form of a surd is $\sqrt[n]{a}$

$\sqrt{\ }$ is called the *radical sign*

n is called the *order* of the radical.

a is called the *radicand*.



1.2.1 Index Form of a Surd

The index form of a surd $\sqrt[n]{a}$ is $a^{\frac{1}{n}}$

Think and Answer !
$a^{\frac{1}{3}}$ and a^3 differ. why?

For example, $\sqrt[5]{8}$ can be written in index form as $\sqrt[5]{8} = (8)^{\frac{1}{5}}$

In the following table, the index form, order and radicand of some surds are given.

Surd	Index Form	Order	Radicand
$\sqrt{5}$	$5^{\frac{1}{2}}$	2	5
$\sqrt[3]{14}$	$(14)^{\frac{1}{3}}$	3	14
$\sqrt[4]{7}$	$7^{\frac{1}{4}}$	4	7
$\sqrt{50}$	$(50)^{\frac{1}{2}}$	2	50
$\sqrt[5]{11}$	$(11)^{\frac{1}{5}}$	5	11

Remark If $\sqrt[n]{a}$ is a surd, then

(i) a is a positive rational number. (ii) $\sqrt[n]{a}$ is an irrational number.

In the table given below both the columns A and B have irrational numbers.

A	B
$\sqrt{5}$	$\sqrt{2+\sqrt{3}}$
$\sqrt[3]{7}$	$\sqrt[3]{5+\sqrt{7}}$
$\sqrt[3]{100}$	$\sqrt[3]{10-\sqrt[3]{3}}$
$\sqrt{12}$	$\sqrt[4]{15+\sqrt{5}}$

The numbers in Column A are surds and the numbers in Column B are irrationals.

Thus, every surd is an irrational number, but every irrational number need not be a surd.

1.2.2 Reduction of a Surd to its Simplest Form

We can reduce a surd to its simplest form.

For example, consider the surd $\sqrt{50}$

Now $\sqrt{50} = \sqrt{25\times 2} = \sqrt{25}\,\sqrt{2} = \sqrt{5^2}\,\sqrt{2} = 5\sqrt{2}$

Thus $5\sqrt{2}$ is the simplest form of $\sqrt{50}$.

1.2.3 Like and Unlike Surds

Surds in their simplest form are called like surds if their order and radicand are the same. Otherwise the surds are called unlike surds.

For example,

(i) $\sqrt{5}$, $4\sqrt{5}$, $-6\sqrt{5}$ are like surds. (ii) $\sqrt{10}$, $\sqrt[3]{3}$, $\sqrt[4]{5}$, $\sqrt[3]{81}$ are unlike surds.

1.2.4 Pure surds

A Surd is called a pure surd if its rational coefficient is unity

For example, $\sqrt{3}$, $\sqrt[3]{5}$, $\sqrt[4]{12}$, $\sqrt{80}$ are pure surds.

1.2.5 Mixed Surds

A Surd is called a mixed if its rational coefficient is other than unity

For example, $2\sqrt{3}$, $5\sqrt[3]{5}$, $3\sqrt[4]{12}$ are mixed surds.

A mixed surd can be converted into a pure surd and a pure surd may or may not be converted into a mixed surd.

For example,

(i) $\sqrt{80} = \sqrt{16\times 5} = 4\sqrt{5}$ (ii) $3\sqrt{2} = \sqrt{3^2\times 2} = \sqrt{9\times 2} = \sqrt{18}$

(iii) $\sqrt{17}$ is a pure surd, but it cannot be converted into a mixed surd.

Laws of Radicals

For positive integers m, n and positive rational numbers a, b we have

(i) $(\sqrt[n]{a})^n = a = \sqrt[n]{a^n}$ (ii) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

(iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$ (iv) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

Using (i) we have $(\sqrt{a})^2 = a$, $\sqrt[3]{a^3} = (\sqrt[3]{a})^3 = a$

Example 1.1

Convert the following surds into index form.

(i) $\sqrt{7}$ (ii) $\sqrt[4]{8}$ (iii) $\sqrt[3]{6}$ (iv) $\sqrt[8]{12}$

Solution In index form we write the given surds as follows

(i) $\sqrt{7} = 7^{\frac{1}{2}}$ (ii) $\sqrt[4]{8} = 8^{\frac{1}{4}}$ (iii) $\sqrt[3]{6} = 6^{\frac{1}{3}}$ (iv) $\sqrt[8]{12} = (12)^{\frac{1}{8}}$

Example 1.2

Express the following surds in its simplest form.

(i) $\sqrt[3]{32}$ (ii) $\sqrt{63}$ (iii) $\sqrt{243}$ (iv) $\sqrt[3]{256}$

Solution

(i) $\sqrt[3]{32} = \sqrt[3]{8\times4} = \sqrt[3]{8}\times\sqrt[3]{4} = \sqrt[3]{2^3}\times\sqrt[3]{4} = 2\sqrt[3]{4}$

(ii) $\sqrt{63} = \sqrt{9\times7} = \sqrt{9}\times\sqrt{7} = 3\sqrt{7}$

(iii) $\sqrt{243} = \sqrt{81\times3} = \sqrt{81}\times\sqrt{3} = \sqrt{9^2}\times\sqrt{3} = 9\sqrt{3}$

(iv) $\sqrt[3]{256} = \sqrt[3]{64\times4} = \sqrt[3]{64}\times\sqrt[3]{4} = \sqrt[3]{4^3}\times\sqrt[3]{4} = 4\sqrt[3]{4}$

Example 1.3

Express the following mixed surds into pure surds.

(i) $16\sqrt{2}$ (ii) $3\sqrt[3]{2}$ (iii) $2\sqrt[4]{5}$ (iv) $6\sqrt{3}$

Solution

(i) $16\sqrt{2} = \sqrt{16^2} \times \sqrt{2}$ $(\because 16 = \sqrt{16^2})$

$= \sqrt{16^2 \times 2} = \sqrt{256 \times 2} = \sqrt{512}$

(ii) $3\sqrt[3]{2} = \sqrt[3]{3^3 \times 2}$ $(\because 3 = \sqrt[3]{3^3})$

$= \sqrt[3]{27 \times 2} = \sqrt[3]{54}$

(iii) $2\sqrt[4]{5} = \sqrt[4]{2^4 \times 5}$ $(\because 2 = \sqrt[4]{2^4})$

$= \sqrt[4]{16 \times 5} = \sqrt[4]{80}$

(iv) $6\sqrt{3} = \sqrt{6^2 \times 3}$ $(\because 6 = \sqrt{6^2})$

$= \sqrt{36 \times 3} = \sqrt{108}$

Example 1.4

Identify whether $\sqrt{32}$ is rational or irrational.

Solution $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$

4 is a rational number and $\sqrt{2}$ is an irrational number.

$\therefore 4\sqrt{2}$ is an irrational number and hence $\sqrt{32}$ is an irrational number.

Example 1.5

Identify whether the following numbers are rational or irrational.

(i) $3 + \sqrt{3}$ (ii) $(4 + \sqrt{2}) - (4 - \sqrt{3})$ (iii) $\dfrac{\sqrt{18}}{2\sqrt{2}}$ (iv) $\sqrt{19} - (2 + \sqrt{19})$

(v) $\dfrac{2}{\sqrt{3}}$ (vi) $\sqrt{12} \times \sqrt{3}$

Solution

(i) $3+\sqrt{3}$

3 is a rational number and $\sqrt{3}$ is irrational. Hence, $3+\sqrt{3}$ is irrational.

(ii) $(4+\sqrt{2})-(4-\sqrt{3})$

$= 4+\sqrt{2}-4+\sqrt{3} = \sqrt{2}+\sqrt{3}$, is irrational.

(iii) $\frac{\sqrt{18}}{2\sqrt{2}} = \frac{\sqrt{9\times 2}}{2\sqrt{2}} = \frac{\sqrt{9}\times\sqrt{2}}{2\sqrt{2}} = \frac{3}{2}$, is rational.

(iv) $\sqrt{19}-(2+\sqrt{19}) = \sqrt{19}-2-\sqrt{19} = -2$, is rational.

(v) $\frac{2}{\sqrt{3}}$ here 2 is rational and $\sqrt{3}$ is irrational. Hence, $\frac{2}{\sqrt{3}}$ is irrational.

(vi) $\sqrt{12}\times\sqrt{3} = \sqrt{12\times 3} = \sqrt{36} = 6$, is rational.

1.3 Four Basic Operations on Surds

1.3.1 Addition and Subtraction of Surds

Like surds can be added and subtracted.

Example 1.6

Simplify

(i) $10\sqrt{2}-2\sqrt{2}+4\sqrt{32}$

(ii) $\sqrt{48}-3\sqrt{72}-\sqrt{27}+5\sqrt{18}$

(iii) $\sqrt[3]{16}+8\sqrt[3]{54}-\sqrt[3]{128}$

Solution

(i) $10\sqrt{2} - 2\sqrt{2} + 4\sqrt{32}$

$= 10\sqrt{2} - 2\sqrt{2} + 4\sqrt{16\times 2}$

$= 10\sqrt{2} - 2\sqrt{2} + 4\times 4\times\sqrt{2}$

$= (10 - 2 + 16)\sqrt{2} = 24\sqrt{2}$

(ii) $\sqrt{48} - 3\sqrt{72} - \sqrt{27} + 5\sqrt{18}$

$= \sqrt{16\times 3} - 3\sqrt{36\times 2} - \sqrt{9\times 3} + 5\sqrt{9\times 2}$

$= \sqrt{16}\sqrt{3} - 3\sqrt{36}\sqrt{2} - \sqrt{9}\sqrt{3} + 5\sqrt{9}\sqrt{2}$

$= 4\sqrt{3} - 18\sqrt{2} - 3\sqrt{3} + 15\sqrt{2}$

$= (-18 + 15)\sqrt{2} + (4 - 3)\sqrt{3} = -3\sqrt{2} + \sqrt{3}$

(iii) $\sqrt[3]{16} + 8\sqrt[3]{54} - \sqrt[3]{128}$

$= \sqrt[3]{8\times 2} + 8\sqrt[3]{27\times 2} - \sqrt[3]{64\times 2}$

$= \sqrt[3]{8}\sqrt[3]{2} + 8\sqrt[3]{27}\sqrt[3]{2} - \sqrt[3]{64}\sqrt[3]{2}$

$= 2\sqrt[3]{2} + 8\times 3\times\sqrt[3]{2} - 4\sqrt[3]{2}$

$= 2\sqrt[3]{2} + 24\sqrt[3]{2} - 4\sqrt[3]{2}$

$= (2 + 24 - 4)\sqrt[3]{2} = 22\sqrt[3]{2}$

1.3.2 Multiplication of Surds

Product of two like surds can simplified using the following law.

$$\sqrt[n]{a}\times\sqrt[n]{b} = \sqrt[n]{ab}$$

Example 1.7

Multiply (i) $\sqrt[3]{13}\times\sqrt[3]{5}$ (ii) $\sqrt[4]{32}\times\sqrt[4]{8}$

Solution

(i) $\sqrt[3]{13}\times\sqrt[3]{5} = \sqrt[3]{13\times 5} = \sqrt[3]{65}$

(ii) $\sqrt[4]{32}\times\sqrt[4]{8} = \sqrt[4]{32\times 8}$

$= \sqrt[4]{2^5\times 2^3} = \sqrt[4]{2^8} = \sqrt[4]{2^4\times 2^4} = 2\times 2 = 4$

1.3.3 Division of Surds

Like surds can be divided using the law

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Example 1.8

Simplify (i) $15\sqrt{54} \div 3\sqrt{6}$ (ii) $\sqrt[3]{128} \div \sqrt[3]{64}$

Solution

(i) $15\sqrt{54} \div 3\sqrt{6}$

$$= \frac{15\sqrt{54}}{3\sqrt{6}} = 5\sqrt{\frac{54}{6}} = 5\sqrt{9} = 5 \times 3 = 15$$

(ii) $\sqrt[3]{128} \div \sqrt[3]{64}$

$$= \frac{\sqrt[3]{128}}{\sqrt[3]{64}} = \sqrt[3]{\frac{128}{64}} = \sqrt[3]{2}$$

Note When the order of the surds are different, we convert them to the same order and then multiplication or division is carried out.

Result $\sqrt[n]{a} = \sqrt[m]{a^{\frac{m}{n}}}$

For example, (i) $\sqrt[3]{5} = \sqrt[12]{5^{\frac{12}{3}}} = \sqrt[12]{5^4}$ (ii) $\sqrt[4]{11} = \sqrt[8]{11^{\frac{8}{4}}} = \sqrt[8]{11^2}$

1.3.4 Comparison of Surds

Irrational numbers of the same order can be compared. Among the irrational numbers of same order, the greatest irrational number is the one with the largest radicand.

If the order of the irrational numbers are not the same, we first convert them to the same order. Then, we just compare the radicands.

Example 1.9

Convert the irrational numbers $\sqrt{3}$, $\sqrt[3]{4}$, $\sqrt[4]{5}$ to the same order.

Solution The orders of the given irrational numbers are 2, 3 and 4.

LCM of 2, 3 and 4 is 12

$\sqrt{3} = \sqrt[12]{3^6} = \sqrt[12]{729}$

$\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$

$\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$

Example 1.10

Which is greater ? $\sqrt[4]{5}$ or $\sqrt[3]{4}$

Solution The orders of the given irrational numbers are 3 and 4.

We have to convert each of the irrational number to an irrational number of the same order.

LCM of 3 and 4 is 12. Now we convert each irrational number as of order 12.

$\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$

$\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$

$\therefore \sqrt[12]{256} > \sqrt[12]{125} \quad \Rightarrow \sqrt[3]{4} > \sqrt[4]{5}$

Example 1.11

Write the irrational numbers $\sqrt[3]{2}$, $\sqrt[4]{4}$, $\sqrt{3}$ in

(i) ascending order (ii) descending order

Solution The orders of the irrational numbers $\sqrt[3]{2}, \sqrt[4]{4}$ and $\sqrt{3}$ are 3, 4 and 2 respectively LCM of 2, 3, and 4 is 12. Now, we convert each irrational number as of order 12.

$\sqrt[3]{2} = \sqrt[12]{2^4} = \sqrt[12]{16}$

$\sqrt[4]{4} = \sqrt[12]{4^3} = \sqrt[12]{64}$

$\sqrt{3} = \sqrt[12]{3^6} = \sqrt[12]{729}$

$\therefore$ Ascending order: $\sqrt[3]{2}, \sqrt[4]{4}, \sqrt{3}$

Descending order: $\sqrt{3}, \sqrt[4]{4}, \sqrt[3]{2}$.

Exercise 1.1

1. Identify which of the following are surds and which are not with reasons.

   (i) $\sqrt{8}\times\sqrt{6}$ (ii) $\sqrt{90}$ (iii) $\sqrt{180}\times\sqrt{5}$ (iv) $4\sqrt{5}\div\sqrt{8}$ (v) $\sqrt[3]{4}\times\sqrt[3]{16}$

2. Simplify

   (i) $(10+\sqrt{3})(2+\sqrt{5})$ (ii) $(\sqrt{5}+\sqrt{3})^2$

   (iii) $(\sqrt{13}-\sqrt{2})(\sqrt{13}+\sqrt{2})$ (iv) $(8+\sqrt{3})(8-\sqrt{3})$

3. Simplify the following.

   (i) $5\sqrt{75}+8\sqrt{108}-\frac{1}{2}\sqrt{48}$ (ii) $7\sqrt[3]{2}+6\sqrt[3]{16}-\sqrt[3]{54}$

   (iii) $4\sqrt{72}-\sqrt{50}-7\sqrt{128}$ (iv) $2\sqrt[3]{40}+3\sqrt[3]{625}-4\sqrt[3]{320}$

4. Express the following surds in its simplest form.

   (i) $\sqrt[3]{108}$ (ii) $\sqrt{98}$ (iii) $\sqrt{192}$ (iv) $\sqrt[3]{625}$

5. Express the following as pure surds.

   (i) $6\sqrt{5}$ (ii) $5\sqrt[3]{4}$ (iii) $3\sqrt[4]{5}$ (iv) $\frac{3}{4}\sqrt{8}$

6. Simplify the following.

   (i) $\sqrt{5}\times\sqrt{18}$ (ii) $\sqrt[3]{7}\times\sqrt[3]{8}$ (iii) $\sqrt[4]{8}\times\sqrt[4]{12}$ (iv) $\sqrt[3]{3}\times\sqrt[6]{5}$

   (v) $3\sqrt{35}\div2\sqrt{7}$ (vi) $\sqrt[4]{48}\div\sqrt[8]{72}$

7. Which is greater ?

   (i) $\sqrt{2}$ or $\sqrt[3]{3}$ (ii) $\sqrt[3]{3}$ or $\sqrt[4]{4}$ (iii) $\sqrt{3}$ or $\sqrt[4]{10}$

8. Arrange in descending and ascending order.

   (i) $\sqrt[4]{5},\sqrt{3},\sqrt[3]{4}$ (ii) $\sqrt[3]{2},\sqrt[3]{4},\sqrt[4]{4}$ (iii) $\sqrt[3]{2},\sqrt[9]{4},\sqrt[6]{3}$

1.4 Rationalization of Surds

Rationalization of Surds

When the denominator of an expression contains a term with a square root or a number under radical sign, the process of converting into an equivalent expression whose denominator is a rational number is called rationalizing the denominator.

If the product of two irrational numbers is rational, then each one is called the **rationalizing factor** of the other.

Let a and b be integers and x, y be positive integers. Then

Remark

(i) $(a+\sqrt{x})$ and $(a-\sqrt{x})$ are rationalizing factors of each other.

(ii) $(a+b\sqrt{x})$ and $(a-b\sqrt{x})$ are rationalizing factors of each other.

(iii) $\sqrt{x}+\sqrt{y}$ and $\sqrt{x}-\sqrt{y}$ are rationalizing factors of each other.

(iv) $a+\sqrt{b}$ is also called the conjugate of $a-\sqrt{b}$ and $a-\sqrt{b}$ is called the conjugate of $a+\sqrt{b}$.

(v) For rationalizing the denominator of a number, we multiply its numerator and denominator by its rationalizing factor.

Example 1.12

Rationalize the denominator of $\frac{2}{\sqrt{3}}$

Solution Multiplying the numerator and denominator of the given number by $\sqrt{3}$, we get

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Example 1.13

Rationalize the denominator of $\frac{1}{5+\sqrt{3}}$

Solution The denominator is $5+\sqrt{3}$. Its conjugate is $5-\sqrt{3}$ or the rationalizing factor is $5-\sqrt{3}$.

$$\frac{1}{5+\sqrt{3}} = \frac{1}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}}$$

$$= \frac{5-\sqrt{3}}{5^2-(\sqrt{3})^2} = \frac{5-\sqrt{3}}{25-3} = \frac{5-\sqrt{3}}{22}$$

Example 1.14

Simplify $\frac{1}{8-2\sqrt{5}}$ by rationalizing the denominator.

Solution Here the denominator is $8-2\sqrt{5}$. The rationalizing factor is $8+2\sqrt{5}$

$$\frac{1}{8-2\sqrt{5}} = \frac{1}{8-2\sqrt{5}} \times \frac{8+2\sqrt{5}}{8+2\sqrt{5}}$$

$$= \frac{8+2\sqrt{5}}{8^2-(2\sqrt{5})^2} = \frac{8+2\sqrt{5}}{64-20}$$

$$= \frac{8+2\sqrt{5}}{44} = \frac{2(4+\sqrt{5})}{44} = \frac{4+\sqrt{5}}{22}$$

Example 1.15

Simplify $\frac{1}{\sqrt{3}+\sqrt{5}}$ by rationalizing the denominator.

Solution Here the denominator is $\sqrt{3}+\sqrt{5}$. So, the rationalizing factor is $\sqrt{3}-\sqrt{5}$

$$\frac{1}{\sqrt{3}+\sqrt{5}} = \frac{1}{\sqrt{3}+\sqrt{5}} \times \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}$$

$$= \frac{\sqrt{3}-\sqrt{5}}{(\sqrt{3})^2-(\sqrt{5})^2} = \frac{\sqrt{3}-\sqrt{5}}{3-5}$$

$$= \frac{\sqrt{3}-\sqrt{5}}{-2} = \frac{\sqrt{5}-\sqrt{3}}{2}$$

Example 1.16

If $\frac{\sqrt{7}-1}{\sqrt{7}+1} + \frac{\sqrt{7}+1}{\sqrt{7}-1} = a + b\sqrt{7}$, find the values of a and b.

Solution

$$\frac{\sqrt{7}-1}{\sqrt{7}+1} + \frac{\sqrt{7}+1}{\sqrt{7}-1} = \frac{\sqrt{7}-1}{\sqrt{7}+1} \times \frac{\sqrt{7}-1}{\sqrt{7}-1} + \frac{\sqrt{7}+1}{\sqrt{7}-1} \times \frac{\sqrt{7}+1}{\sqrt{7}+1}$$

$$= \frac{(\sqrt{7}-1)^2}{(\sqrt{7})^2-1} + \frac{(\sqrt{7}+1)^2}{(\sqrt{7})^2-1}$$

$$= \frac{7+1-2\sqrt{7}}{7-1} + \frac{7+1+2\sqrt{7}}{7-1}$$

$$= \frac{8-2\sqrt{7}}{6} + \frac{8+2\sqrt{7}}{6}$$

$$= \frac{8-2\sqrt{7}+8+2\sqrt{7}}{6}$$

$$= \frac{16}{6} = \frac{8}{3} + 0\sqrt{7}$$

$$\therefore \frac{8}{3} + 0\sqrt{7} = a + b\sqrt{7} \Longrightarrow a = \frac{8}{3},\ b = 0.$$

Exmaple 1.17

If $x = 1 + \sqrt{2}$, find $\left(x - \frac{1}{x}\right)^2$

Solution $x = 1 + \sqrt{2}$

$$\Longrightarrow \quad \frac{1}{x} = \frac{1}{1+\sqrt{2}}$$

$$= \frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$$

$$= \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1} = -(1-\sqrt{2})$$

$$\therefore x - \frac{1}{x} = (1+\sqrt{2}) - \{-(1-\sqrt{2})\}$$

$$= 1 + \sqrt{2} + 1 - \sqrt{2} = 2$$

Hence, $\left(x - \frac{1}{x}\right)^2 = 2^2 = 4.$

- If 'a' is a positive rational number and n is a positive integer such that $\sqrt[n]{a}$ is an irrational number, then $\sqrt[n]{a}$ is called a 'surd' or a 'radical'.
- For positive integers m, n and positive rational numbers a, b we have

  (i) $\left(\sqrt[n]{a}\right)^n = a = \sqrt[n]{a^n}$ (ii) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

  (iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$ (iv) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

- When the denominator of an expression contains a term with a square root or a number under radical sign, the process of converting to an equivalent expression whose denominator is a rational number is called rationalizing the denominator.
- If the product of two irrational numbers is rational, then each one is called the rationalizing factor of the other.
- If a and b are any two positive integers, there exist two non-negative integers q and r such that $a = bq + r,\ 0 \le r < b$. (Division Algorithm)

Exercise 1.4

Multiple Choice Questions

1. Which one of the following is not a surd?

   (A) $\sqrt[3]{8}$ (B) $\sqrt[3]{30}$ (C) $\sqrt[5]{4}$ (D) $\sqrt[8]{3}$

2. The simplest form of $\sqrt{50}$ is

   (A) $5\sqrt{10}$ (B) $5\sqrt{2}$ (C) $10\sqrt{5}$ (D) $25\sqrt{2}$

3. $\sqrt[4]{11}$ is equal to

   (A) $\sqrt[8]{11^2}$ (B) $\sqrt[8]{11^4}$ (C) $\sqrt[8]{11^8}$ (D) $\sqrt[8]{11^6}$

4. $\dfrac{2}{\sqrt{2}}$ is equal to

   (A) $2\sqrt{2}$ (B) $\sqrt{2}$ (C) $\dfrac{\sqrt{2}}{2}$ (D) 2

5. The ratioanlising factor of $\dfrac{5}{\sqrt[3]{3}}$ is

   (A) $\sqrt[3]{6}$ (B) $\sqrt[3]{3}$ (C) $\sqrt[3]{9}$ (D) $\sqrt[3]{27}$

Key Concept **Scientific Notation**

A number N is in *scientific notation* when it is expressed as the product of a decimal number between 1 and 10 and some integral power of 10.

$$N = a \times 10^n, \text{ where } 1 \le a < 10 \text{ and } n \text{ is an integer.}$$

To transform numbers from decimal notation to scientific notation, the laws of exponents form the basis for calculations using powers. Let m and n be natural numbers and a is a real number. The laws of exponents are given below:

(i) $a^m \times a^n = a^{m+n}$ (Product law)

(ii) $\dfrac{a^m}{a^n} = a^{m-n}$ (Quotient law)

(iii) $(a^m)^n = a^{mn}$ (Power law)

(iv) $a^m \times b^m = (a \times b)^m$ (Combination law)

For $a \neq 0$, we define $a^{-m} = \dfrac{1}{a^m}$, and $a^0 = 1$.

Example 2.1

Express 9781 in scientific notation.

Solution In integers, the decimal point at the end is usually omitted.

$$9\ 7\ 8\ 1\ .$$
$$\ \ 3\ \ 2\ \ 1$$

The decimal point is to be moved 3 places to the left of its original position. So the power of 10 is 3.

$\therefore\ 9781 = 9.781 \times 10^3$

Example 2.2

Express $0 \cdot 000432078$ in scientific notation.

Solution $0\ .\ 0\ 0\ 0\ 4\ 3\ 2\ 0\ 7\ 8$

$$1\ \ 2\ \ 3\ \ 4$$

The decimal point is to be moved four places to the right of its original position. So the power of 10 is – 4

$\therefore\ 0.000432078 = 4.32078 \times 10^{-4}$

2.2.1 Multiplication and Division in Scientific Notation

One can find the product or quotient of very large(googolplex) or very small numbers easily in scientific notion.

Example 2.5

Write the following in scientific notation.

(i) $(4000000)^3$ (ii) $(5000)^4 \times (200)^3$

(iii) $(0.00003)^5$ (iv) $(2000)^2 \div (0.0001)^4$

Solution

(i) First we write the number (within the brackets) in scientific notation.

$$4000000 = 4.0 \times 10^6$$

Now, raising to the power 3 on both sides we get,

$$\therefore (4000000)^3 = (4.0 \times 10^6)^3 = (4.0)^3 \times (10^6)^3$$
$$= 64 \times 10^{18} \quad = 6.4 \times 10^1 \times 10^{18} = 6.4 \times 10^{19}$$

(ii) In scientific notation,

$5000 = 5.0 \times 10^3$ and $200 = 2.0 \times 10^2$.

$$\therefore (5000)^4 \times (200)^3 = (5.0 \times 10^3)^4 \times (2.0 \times 10^2)^3$$
$$= (5.0)^4 \times (10^3)^4 \times (2.0)^3 \times (10^2)^3$$
$$= 625 \times 10^{12} \times 8 \times 10^6 = 5000 \times 10^{18}$$
$$= 5.0 \times 10^3 \times 10^{18} = 5.0 \times 10^{21}$$

(iii) In scientific notation, $0.00003 = 3.0 \times 10^{-5}$

$$\therefore (0.00003)^5 = (3.0 \times 10^{-5})^5 = (3.0)^5 \times (10^{-5})^5$$
$$= 243 \times 10^{-25} \quad = 2.43 \times 10^2 \times 10^{-25} = 2.43 \times 10^{-23}$$

(iv) In scientific notation,

$2000 = 2.0 \times 10^3$ and $0.0001 = 1.0 \times 10^{-4}$

$$\therefore (2000)^2 \div (0.0001)^4 = \frac{(2.0 \times 10^3)^2}{(1.0 \times 10^{-4})^4} = \frac{(2.0)^2 \times (10^3)^2}{(1.0)^4 \times (10^{-4})^4}$$
$$= \frac{4 \times 10^6}{10^{-16}} = 4.0 \times 10^{6-(-16)} = 4.0 \times 10^{22}$$

Exercise 2.1

1. Represent the following numbers in the scientific notation.

(i) 749300000000 (ii) 13000000 (iii) 105003

(iv) 543600000000000 (v) 0.0096 (vi) 0.0000013307

(vii) 0.0000000022 (viii) 0.0000000000009

2. Write the following numbers in decimal form.

(i) 3.25×10^{-6} (ii) 4.134×10^{-4} (iii) 4.134×10^{4}

(iv) 1.86×10^{7} (v) 9.87×10^{9} (vi) 1.432×10^{-9}

3. Represent the following numbers in scientific notation.

(i) $(1000)^2 \times (20)^6$ (ii) $(1500)^3 \times (0.0001)^2$

(iii) $(16000)^3 \div (200)^4$ (iv) $(0.003)^7 \times (0.0002)^5 \div (0.001)^3$

(v) $(11000)^3 \times (0.003)^2 \div (30000)$

Key Concept **Sector**

A sector is the part of a circle enclosed by any two radii of the circle and their intercepted arc.

4.2.1 Central Angle or Sector Angle of a Sector

Key Concept **Central Angle**

Central Angle is the angle subtended by the arc of the sector at the centre of the circle in which the sector forms a part.

In fig.4.2, the angle subtended by the arc $\overset{\frown}{PQ}$ at the centre is θ. So the central angle of the sector POQ is θ.

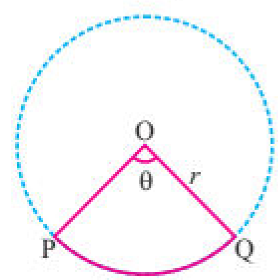


Fig. 4.2

For example,

1. A semi- circle is a sector whose central angle is 180°.
2. A quadrant of a circle is a sector whose central angle is 90°.

4.2.2 Length of Arc (Arc Length) of a Sector

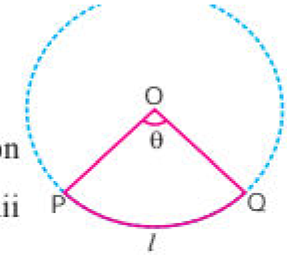


Fig. 4.3

In fig.4.3, arc length of a sector POQ is the length of the portion on the circumference of the circle intercepted between the bounding radii (OP and OQ) and is denoted by l.

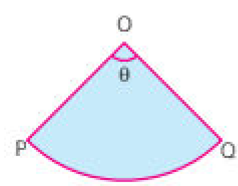
For example,

1. Length of arc of a circle is its circumference. i.e., $l=2\pi r$ units, where r is the radius.
2. Length of arc of a semicircle is $l=2\pi r\times\frac{180^\circ}{360^\circ}=\pi r$ units, where r is the radius and the central angle is 180°.
3. Length of arc of a quadrant of a circle is $l=2\pi r\times\frac{90^\circ}{360^\circ}=\frac{\pi r}{2}$ units, where r is the radius and the central angle is 90°.

Key Concept	**Length of Arc**
If θ is the central angle and r is the radius of a sector, then its arc length is given by $l=\frac{\theta}{360^\circ}\times 2\pi r$ units	

4.2.3 Area of a Sector

Area of a sector is the region bounded by the bounding radii and the arc of the sector.



For Example,

1. Area of a circle is πr^2 square units.
2. Area of a semicircle is $\frac{\pi r^2}{2}$ square units.
3. Area of a quadrant of a circle is $\frac{\pi r^2}{4}$ square units.

Key Concept	**Area of a Sector**
If θ is the central angle and r is the radius of a sector, then the area of the sector is $\frac{\theta}{360^\circ}\times\pi r^2$ square units.	

Let us find the relationship between area of a sector, its arc length l and radius r.

$$\text{Area} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{\theta}{360^\circ} \times \frac{2\pi r}{2} \times r$$

$$= \frac{1}{2} \times \left(\frac{\theta}{360^\circ} \times 2\pi r\right) \times r$$

$$= \frac{1}{2} \times lr$$

$$\boxed{\text{Area of sector} = \frac{lr}{2} \text{ square units.}}$$

4.2.4 Perimeter of a Sector

The perimeter of a sector is the sum of the lengths of all its boundaries. Thus, perimeter of a sector is $l + 2r$ units.

Key Concept **Perimeter of a Sector**

If l is the arc length and r is the radius of a sector, then its perimeter P is given by the formula $P = l + 2r$ units.

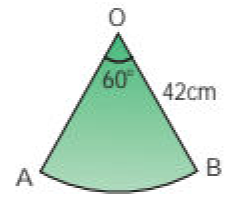
For example,

1. Perimeter of a semicircle is $(\pi + 2)r$ units.
2. Perimeter of a quadrant of a circle is $\left(\frac{\pi}{2} + 2\right)r$ units.

Note
1. Length of an arc and area of a sector are proportional to the central angle.
2. As π is an irrational number, we use its approximate val1ue $\frac{22}{7}$ or 3.14 in our calculations.

Example 4.1

The radius of a sector is 42 cm and its sector angle is 60°. Find its arc length, area and perimeter.



Solution Given that radius $r = 42$ cm and $\theta = 60^\circ$. Therefore,

$$\text{length of arc } l = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 42 = 44 \text{ cm.}$$

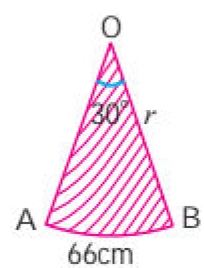
$$\text{Area of the sector} = \frac{lr}{2} = \frac{44 \times 42}{2} = 924 \text{ cm}^2.$$

$$\text{Perimeter} = l + 2r$$

$$= 44 + 2(42) = 128 \text{ cm.}$$

Example 4.2

The arc length of a sector is 66 cm and the central angle is 30°. Find its radius.



Solution Given that $\theta = 30^\circ$ and $l = 66$ cm. So,

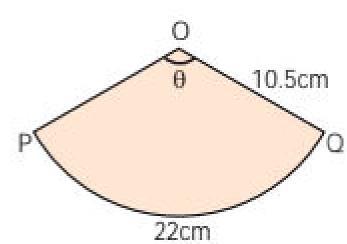
$$\frac{\theta}{360^\circ} \times 2\pi r = l$$

$$\text{i. e.,} \frac{30^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times r = 66$$

$$\therefore r = 66 \times \frac{360^\circ}{30^\circ} \times \frac{1}{2} \times \frac{7}{22} = 126 \text{ cm}$$

Example 4.3

The length of arc of a sector is 22 cm and its radius is 10.5 cm. Find its central angle.



Solution Given that $r = 10.5$ cm and $l = 22$ cm.

$$\frac{\theta}{360^\circ} \times 2\pi r = l$$

$$\text{i. e.,} \frac{\theta}{360^\circ} \times 2 \times \frac{22}{7} \times 10.5 = 22$$

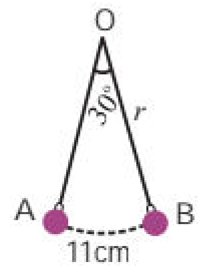
$$\therefore \theta = 22 \times 360^\circ \times \frac{1}{2} \times \frac{7}{22} \times \frac{1}{10.5} = 120^\circ$$

Example 4.4

A pendulum swings through an angle of 30° and describes an arc length of 11 cm. Find the length of the pendulum.

Solution If the pendulum swings once, then it forms a sector and the radius of the sector is the length of the pendulum. So,

$\theta = 30^\circ$, $l = 11$ cm



Using the formula $\frac{\theta}{360^\circ} \times 2\pi r = l$, we have

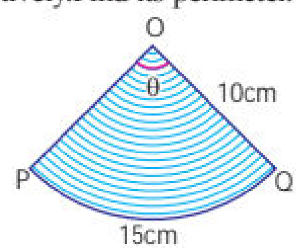
$$\frac{30^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times r = 11$$

$$\therefore\ r = 11 \times \frac{360^\circ}{30^\circ} \times \frac{1}{2} \times \frac{7}{22} = 21 \text{ cm}$$

Example 4.5

The radius and length of arc of a sector are 10 cm and 15 cm respectively.Find its perimeter.

Solution Given that $r = 10$ cm, $l = 15$ cm



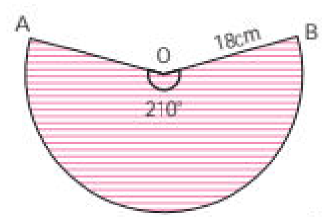
Perimeter of the sector $= l + 2r = 15 + 2(10)$

$= 15 + 20 = 35$ cm

Example 4.6

Find the perimeter of a sector whose radius and central angle are 18 cm and 210° respectively.

Solution Given that $r = 18$ cm, $\theta = 210^\circ$. Hence,



$$l = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{210^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 18 = 66 \text{ cm}$$

$\therefore$ Perimeter of the sector $= l + 2r = 66 + 2(18) = 66 + 36 = 102$ cm

Example 4.7

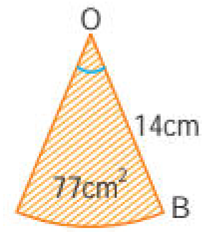
The area of a sector of a circle of radius 14 cm is 77 cm^2. Find its central angle.

Solution Given that $r = 14$ cm, area $= 77$ cm^2

$$\frac{\theta}{360^\circ} \times \pi r^2 = \text{Area of the sector}$$

$$\frac{\theta}{360^\circ} \times \frac{22}{7} \times 14 \times 14 = 77$$

$$\therefore\ \theta = \frac{77 \times 360^\circ \times 7}{22 \times 14 \times 14} = 45^\circ$$



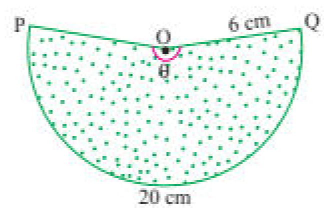
Example 4.8

Calculate the area of a sector whose radius and arc length are 6 cm and 20 cm respectively.

Solution Given that $r = 6$ cm, $l = 20$ cm

$$\text{Area} = \frac{lr}{2} \text{ square units}$$

$$= \frac{20 \times 6}{2} = 60 \text{ cm}^2$$



Example 4.9

If the perimeter and radius of a sector are 38 cm and 9 cm respectively, find its area.

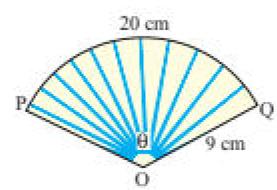
Solution Given, $r = 9$ cm, perimeter $= 38$ cm

$$\text{Perimeter} = l + 2r = 38$$

$$\text{i.e.,} \quad l + 18 = 38$$

$$l = 38 - 18 = 20 \text{ cm}$$

$$\therefore \text{Area} = \frac{lr}{2} = \frac{20 \times 9}{2} = 90 \text{ cm}^2$$



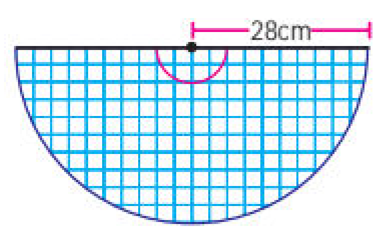
Example 4.10

Find the perimeter and area of a semicircle of radius 28 cm.

Solution Given, $r = 28$ cm

$$\text{Perimeter} = (\pi + 2)\, r = \left(\frac{22}{7} + 2\right) 28 = 144 \text{ cm}$$

$$\text{Area} = \frac{\pi r^2}{2} = \frac{22}{7} \times \frac{28 \times 28}{2} = 1232 \text{ cm}^2$$



Example 4.11

Find the radius, central angle and perimeter of a sector whose arc length and area are 27.5 cm and 618.75 cm² respectively.

Solution Given that l = 27.5 cm and Area = 618.75 cm². So,

$$\text{Area} = \frac{lr}{2} = 618.75 \text{ cm}^2$$

$$\text{i.e.} \frac{27.5 \times r}{2} = 618.75$$

$$\therefore r = 45 \text{ cm}$$

Hence, perimeter is $l + 2r = 27.5 + 2(45) = 117.5$cm

Now, arc length is given by $\frac{\theta}{360^\circ} \times 2\pi r = l$

$$\text{i.e. } \frac{\theta}{360^\circ} \times 2 \times \frac{22}{7} \times 45 = 27.5$$

$$\therefore \theta = 35^\circ$$

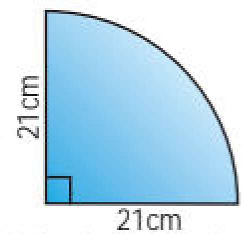
Example 4.12

Calculate the area and perimeter of a quadrant of a circle of radius 21 cm.

Solution Given that $r = 21$ cm, $\theta = 90^\circ$

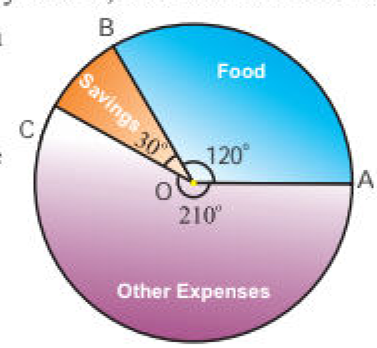
$$\text{Perimeter} = \left(\frac{\pi}{2} + 2\right)r = \left(\frac{22}{7 \times 2} + 2\right) \times 21 = 75 \text{ cm}$$

$$\text{Area} = \frac{\pi r^2}{4} = \frac{22}{7 \times 4} \times 21 \times 21 = 346.5 \text{ cm}^2$$



Example 4.13

Monthly expenditure of a person whose monthly salary is ₹ 9,000 is shown in the adjoining figure. Find the amount he has (i) spent for food (ii) in his savings



Solution Let ₹ 9,000 be represented by the area of the circle, i. e., $\pi r^2 = 9000$

(i) Area of sector $AOB = \frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{120^\circ}{360^\circ} \times 9000 = 3,000$$

Amount spent for food is ₹ 3,000.

(ii) Area of sector $BOC = \frac{\theta}{360^\circ} \times \pi r^2$

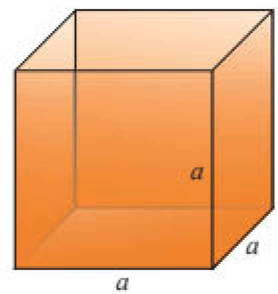
$$= \frac{30^\circ}{360^\circ} \times 9,000 = 750$$

Amount saved in savings is ₹ 750.

4.3.1 Surface Area of a Cube

The sum of the areas of all the six equal faces is called the *Total Surface Area* (T.S.A) of the cube.

In the adjoining figure, let the side of the cube measure a units. Then the area of each face of the cube is a^2 square units. Hence, the total surface area is $6a^2$ square units.



In a cube, if we don't consider the top and bottom faces, the remaining area is called the *Lateral Surface Area* (L.S.A). Hence, the lateral surface area of the cube is $4a^2$ square units.

Key Concept **Surface Area of Cube**

Let the side of a cube be a units. Then:

(i) The Total Surface Area (T.S.A) = $6a^2$ square units.

(ii) The Lateral Surface Area (L.S.A) = $4a^2$ square units.

4.3.2 Volume of a Cube

Key Concept **Volume of Cube**

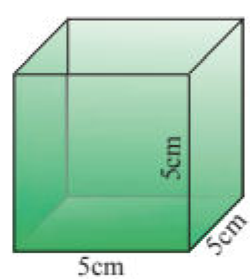
If the side of a cube is a units, then its volume V is given by the formula

$$V = a^3 \text{ cubic units}$$

Note Volume can also be defined as the number of unit cubes required to fill the entire cube.

Example 4.16

Find the L.S.A, T.S.A and volume of a cube of side 5 cm.



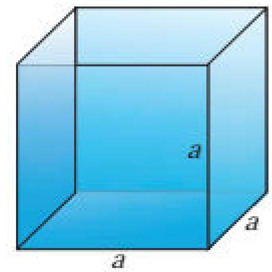
Solution L.S.A $= 4a^2 = 4(5^2) = 100$ sq. cm

T.S.A $= 6a^2 = 6\,(5^2) = 150$ sq. cm

Volume $= a^3 = 5^3 = 125$ cm³

Example: 4.17

Find the length of the side of a cube whose total surface area is 216 square cm.



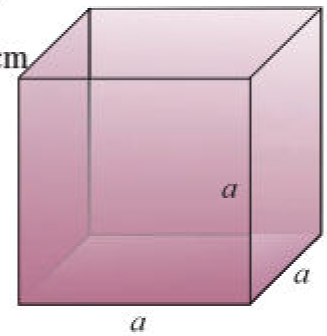
Solution Let a be the side of the cube. Given that T.S.A = 216 sq. cm

i. e., $6a^2 = 216 \implies a^2 = \frac{216}{6} = 36$

$\therefore \quad a = \sqrt{36} = 6$ cm

Example 4.18

A cube has a total surface area of 384 sq. cm. Find its volume.



Solution Let a be the side of the cube. Given that T.S.A = 384 sq. cm

$6a^2 = 384 \implies a^2 = \frac{384}{6} = 64$

$\therefore \; a = \sqrt{64} = 8$ cm

Hence, Volume $= a^3 = 8^3 = 512$ cm^3

Example 4.19

A cubical tank can hold 27,000 litres of water. Find the dimension of its side.

Solution Let a be the side of the cubical tank. Volume of the tank is 27,000 litres. So,

$V = a^3 = \frac{27,000}{1,000} m^3 = 27\, m^3 \qquad \therefore a = \sqrt[3]{27} = 3\, m$

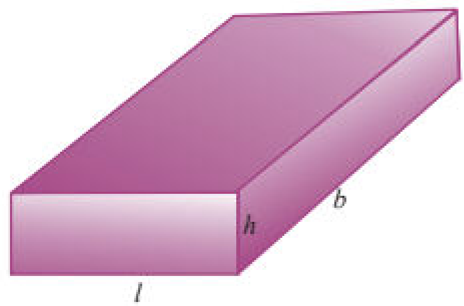
4.4 Cuboids

A cuboid is a three dimensional solid having six rectangular faces.

Example: Bricks, Books etc.,

4.4.1 Surface Area of a Cuboid

Let l, b and h be the length, breadth and height of a cuboid respectively. To find the total surface area, we split the faces into three pairs.



(i) The total area of the front and back faces is $lh + lh = 2lh$ square units.

(ii) The total area of the side faces is $bh + bh = 2bh$ square units.

(iii) The total area of the top and bottom faces is $lb + lb = 2lb$ square units.

The Lateral Surface Area (L.S.A) = $2(l + b)h$ square units.

The Total Surface Area (T.S.A) = $2(lb + bh + lh)$ square units.

Key Concept **Surface Area of a Cuboid**

Let l, b and h be the length, breadth and height of a cuboid respectively. Then

(i) The Lateral Surface Area (L.S.A) = $2(l + b)h$ square units

(ii) The Total Surface Area (T.S.A) = $2(lb + bh + lh)$ sq. units

Note L.S.A. is also equal to the product of the perimeter of the base and the height.

4.4.2 Volume of a Cuboid

Key Concept **Volume of a Cuboid**

If the length, breadth and height of a cuboid are l, b and h respectively, then the volume V of the cuboid is given by the formula

$V = l \times b \times h$ cu. units

Example: 4.20

Find the total surface area of a cuboid whose length, breadth and height are 20 cm, 12 cm and 9 cm respectively.

Solution

Given that $l = 20$ cm, $b = 12$ cm, $h = 9$ cm

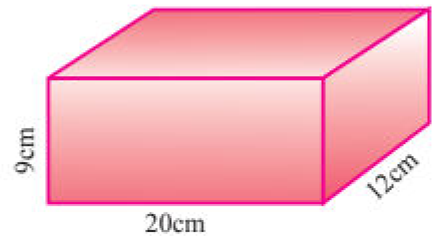
$\therefore$ T.S.A $= 2(lb + bh + lh)$

$= 2[(20\times 12)+(12\times 9)+(20\times 9)]$

$= 2(240 + 108 + 180)$

$= 2\times 528$

$= 1056\ cm^2$



Example: 4.21

Find the L.S.A of a cuboid whose dimensions are given by $3m\times 5m\times 4m$.

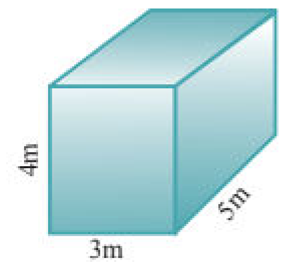
Solution Given that $l = 3$ m, b = 5 m, h = 4 m

L.S.A $= 2(l+b)h$

$= 2\times(3+5)\times 4$

$= 2\times 8\times 4$

$= 64$ sq. m



Example: 4.22

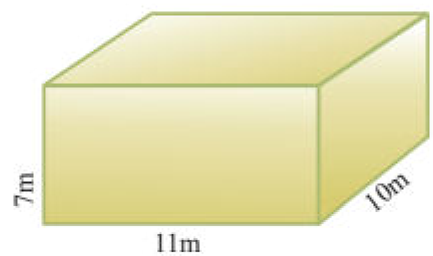
Find the volume of a cuboid whose dimensions are given by 11 m, 10 m and 7 m.

Solution Given that $l = 11$ m, b = 10 m, h = 7 m

volume $= lbh$

$= 11\times 10\times 7$

$= 770$ cu.m.



Example: 4.23

Two cubes each of volume 216 cm^3 are joined to form a cuboid as shown in the figure. Find the T.S.A of the resulting cuboid.

Solution Let the side of each cube be a. Then $a^3 = 216$

$\therefore a = \sqrt[3]{216} = 6$ cm

Now the two cubes of side 6 cm are joined to form a cuboid. So,

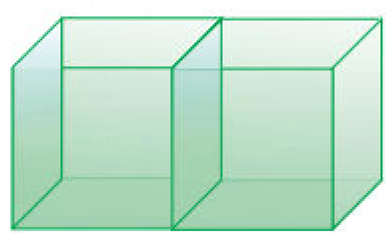
$\therefore l = 6 + 6 = 12$ cm, $b = 6$ cm, $h = 6$ cm

$\therefore$ T.S.A $= 2(lb + bh + lh)$

$= 2[(12\times 6)+(6\times 6)+(12\times 6)]$

$= 2[72 + 36 + 72]$

$= 2\times 180 = 360\ cm^2$



- A sector is the part of a circle enclosed by any two radii of the circle and their intercepted arc.
- Central Angle is the angle subtended by the arc of the sector at the centre of the circle in which the sector forms a part.
- If θ is the central angle and r is the radius of a sector, then its arc length is given by $l = \frac{\theta}{360} \times 2\pi r$ units
- If θ is the central angle and r is the radius of a sector, then the area of the sector is $\frac{\theta}{360} \times \pi r^2$ square units.
- If l is the arc length and r is the radius of a sector, then its perimeter P is given by the formula $P = l + 2r$ units.

- Let the side of a cube be a units. Then:
  (i) The Total Surface Area (T.S.A) = $6a^2$ square units.
  (ii) The Lateral Surface Area (L.S.A) = $4a^2$ square units.
- If the side of a cube is a units, then its volume V is given by the formula, $V = a^3$ cubic units
- Let l, b and h be the length, breadth and height of a cuboid respectively. Then:
  (i) The Lateral Surface Area (L.S.A) = $2(l + b)h$ squre units
  (ii) The Total Surface Area (T.S.A) = $2(lb + bh + lh)$ sq. units
- If the length, breadth and height of a cuboid are l, b and h respectively, then the volume V of the cuboid is given by the formula $V = l \times b \times h$ cu. units

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10th Std

(i) $2, 4, 6, 8, \cdots, 2010.$ (finite number of terms)

(ii) $1, -1, 1, -1, 1, -1, 1, \cdots .$ (terms just keep oscillating between 1 and -1)

(iii) $\pi, \pi, \pi, \pi, \pi.$ (terms are same; such sequences are constant sequences)

(iv) $2, 3, 5, 7, 11, 13, 17, 19, 23, \cdots .$ (list of all prime numbers)

(v) $0.3, 0.33, 0.333, 0.3333, 0.33333, \cdots .$ (infinite number of terms)

(vi) $S = \{a_n\}_1^\infty$ where $a_n = 1$ or 0 according to the outcome head or tail in the n^{th} toss of a coin.

Example 2.1

Write the first three terms in a sequence whose n^{th} term is given by

$$c_n = \frac{n(n+1)(2n+1)}{6}, \forall n \in \mathbb{N}$$

Solution Here, $c_n = \frac{n(n+1)(2n+1)}{6}, \forall n \in \mathbb{N}$

For $n = 1$, $c_1 = \frac{1(1+1)(2(1)+1)}{6} = 1.$

For $n = 2$, $c_2 = \frac{2(2+1)(4+1)}{6} = \frac{2(3)(5)}{6} = 5.$

Finally $n = 3$, $c_3 = \frac{3(3+1)(7)}{6} = \frac{(3)(4)(7)}{6} = 14.$

Hence, the first three terms of the sequence are 1, 5, and 14.

In the above example, we were given a formula for the general term and were able to find any particular term directly. In the following example, we shall see another way of generating a sequence.

Example 2.13

An amount ₹500 is deposited in a bank which pays annual interest at the rate of 10% compounded annually. What will be the value of this deposit at the end of 10^{th} year?

Solution

The principal is ₹500. So, the interest for this principal for one year is $500\left(\frac{10}{100}\right) = 50$.

Thus, the principal for the 2nd year = Principal for 1st year + Interest

$$= 500 + 500\left(\frac{10}{100}\right) = 500\left(1 + \frac{10}{100}\right)$$

Now, the interest for the second year $= \left(500\left(1+\frac{10}{100}\right)\right)\left(\frac{10}{100}\right)$.

So, the principal for the third year $= 500\left(1+\frac{10}{100}\right) + 500\left(1+\frac{10}{100}\right)\frac{10}{100}$

$$= 500\left(1+\frac{10}{100}\right)^2$$

Continuing in this way we see that the principal for the n^{th} year $= 500\left(1+\frac{10}{100}\right)^{n-1}$.

The amount at the end of $(n-1)^{th}$ year = Principal for the n^{th} year.

Thus, the amount in the account at the end of n^{th} year.

$$= 500\left(1+\frac{10}{100}\right)^{n-1} + 500\left(1+\frac{10}{100}\right)^{n-1}\left(\frac{10}{100}\right) = 500\left(\frac{11}{10}\right)^n.$$

The amount in the account at the end of 10^{th} year

$$= ₹\ 500\left(1+\frac{10}{100}\right)^{10} = ₹\ 500\left(\frac{11}{10}\right)^{10}.$$

Remarks

By using the above method, one can derive a formula for finding the total amount for compound interest problems. Derive the formula:

$$A = P(1+i)^n$$

where A is the amount, P is the principal, $i = \frac{r}{100}$, r is the annual interest rate and n is the number of years.

Example 2.16

Find the sum of the arithmetic series $5 + 11 + 17 + \cdots + 95$.

Solution Given that the series $5 + 11 + 17 + \cdots + 95$ is an arithmetic series.

Note that $a = 5$, $d = 11 - 5 = 6$, $l = 95$.

Now, $n = \frac{l-a}{d} + 1$

$$= \frac{95-5}{6} + 1 = \frac{90}{6} + 1 = 16.$$

Hence, the sum $S_n = \frac{n}{2}[l + a]$

$$S_{16} = \frac{16}{2}[95 + 5] = 8(100) = 800.$$

Example 2.17

Find the sum of the first $2n$ terms of the following series.

$$1^2 - 2^2 + 3^2 - 4^2 + \dots.$$

Solution We want to find $1^2 - 2^2 + 3^2 - 4^2 + \cdots$ to $2n$ terms

$$= 1 - 4 + 9 - 16 + 25 - \cdots \text{ to } 2n \text{ terms}$$

$$= (1-4) + (9-16) + (25-36) + \cdots \text{ to } n \text{ terms.} \quad \text{(after grouping)}$$

$$= -3 + (-7) + (-11) + \cdots\ n \text{ terms}$$

Now, the above series is in an A.P. with first term $a = -3$ and common difference $d = -4$

Therefore, the required sum $= \frac{n}{2}[2a + (n-1)d]$

$$= \frac{n}{2}[2(-3) + (n-1)(-4)]$$

$$= \frac{n}{2}[-6 - 4n + 4] = \frac{n}{2}[-4n - 2]$$

$$= \frac{-2n}{2}(2n+1) = -n(2n+1).$$

(i) The sum of the first n natural numbers, $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

(ii) The sum of the first n odd natural numbers, $\sum_{k=1}^{n}(2k-1) = n^2$.

(iii) The sum of first n odd natural numbers (when the last term l is given) is

$$1 + 3 + 5 + \cdots + l = \left(\frac{l+1}{2}\right)^2.$$

(iv) The sum of squares of first n natural numbers,

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

(v) The sum of cubes of the first n natural numbers,

$$\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Example 2.29

Find the sum of the following series

(i) $26 + 27 + 28 + \cdots + 60$ (ii) $1 + 3 + 5 + \cdots$ to 25 terms (iii) $31 + 33 + \cdots + 53$.

Solution

(i) We have $26 + 27 + 28 + \cdots + 60 = (1 + 2 + 3 + \cdots + 60) - (1 + 2 + 3 + \cdots + 25)$

$$= \sum_{1}^{60} n - \sum_{1}^{25} n$$

$$= \frac{60(60+1)}{2} - \frac{25(25+1)}{2}$$

$$= (30 \times 61) - (25 \times 13) = 1830 - 325 = 1505.$$

(ii) Here, $n = 25$

$\therefore$ $1 + 3 + 5 + \cdots$ to 25 terms $= 25^2$ $\quad\left(\sum_{k=1}^{n}(2k-1) = n^2\right)$

$= 625.$

(iii) $31 + 33 + \cdots + 53$

$$= (1 + 3 + 5 + \cdots + 53) - (1 + 3 + 5 + \cdots + 29)$$

$$= \left(\frac{53+1}{2}\right)^2 - \left(\frac{29+1}{2}\right)^2 \qquad \left(1 + 3 + 5 + \cdots + l = \left(\frac{l+1}{2}\right)^2\right)$$

$$= 27^2 - 15^2 = 504.$$

Example 2.30

Find the sum of the following series

(i) $1^2 + 2^2 + 3^2 + \cdots + 25^2$ (ii) $12^2 + 13^2 + 14^2 + \cdots + 35^2$

(iii) $1^2 + 3^2 + 5^2 + \cdots + 51^2$.

Solution

(i) Now, $1^2 + 2^2 + 3^2 + \cdots + 25^2 = \sum_1^{25} n^2$

$$= \frac{25(25+1)(50+1)}{6} \qquad \left(\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \right)$$

$$= \frac{(25)(26)(51)}{6}$$

$\therefore$ $1^2 + 2^2 + 3^2 + \cdots + 25^2 = 5525.$

(ii) Now, $12^2 + 13^2 + 14^2 + \cdots + 35^2$

$$= (1^2 + 2^2 + 3^2 + \cdots + 35^2) - (1^2 + 2^2 + 3^2 + \cdots + 11^2)$$

$$= \sum_1^{35} n^2 - \sum_1^{11} n^2$$

$$= \frac{35(35+1)(70+1)}{6} - \frac{11(12)(23)}{6}$$

$$= \frac{(35)(36)(71)}{6} - \frac{(11)(12)(23)}{6}$$

$$= 14910 - 506 = 14404.$$

(iii) Now, $1^2 + 3^2 + 5^2 + \cdots + 51^2$

$$= (1^2 + 2^2 + 3^2 + \cdots + 51^2) - (2^2 + 4^2 + 6^2 + \cdots 50^2)$$

$$= \sum_1^{51} n^2 - 2^2[1^2 + 2^2 + 3^2 + \cdots + 25^2]$$

$$= \sum_1^{51} n^2 - 4\sum_1^{25} n^2$$

$$= \frac{51(51+1)(102+1)}{6} - 4 \times \frac{25(25+1)(50+1)}{6}$$

$$= \frac{(51)(52)(103)}{6} - 4 \times \frac{25(26)(51)}{6}$$

$$= 45526 - 22100 = 23426.$$

Example 2.31

Find the sum of the series.

(i) $1^3 + 2^3 + 3^3 + \cdots + 20^3$ (ii) $11^3 + 12^3 + 13^3 + \cdots + 28^3$

Solution

(i) $1^3 + 2^3 + 3^3 + \cdots 20^3 = \sum_{1}^{20} n^3$

$$= \left(\frac{20(20+1)}{2}\right)^2 \qquad \text{using } \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

$$= \left(\frac{20 \times 21}{2}\right)^2 = (210)^2 = 44100.$$

(ii) Next we consider $11^3 + 12^3 + \cdots + 28^3$

$$= (1^3 + 2^3 + 3^3 + \cdots + 28^3) - (1^3 + 2^3 + \cdots + 10^3)$$

$$= \sum_{1}^{28} n^3 - \sum_{1}^{10} n^3$$

$$= \left[\frac{28(28+1)}{2}\right]^2 - \left[\frac{10(10+1)}{2}\right]^2$$

$$= 406^2 - 55^2 = (406+55)(406-55)$$

$$= (461)(351) = 161811.$$

Example 2.33

(i) If $1 + 2 + 3 + \cdots + n = 120$, find $1^3 + 2^3 + 3^3 + \cdots n^3$.

(ii) If $1^3 + 2^3 + 3^3 + \cdots + n^3 = 36100$, then find $1 + 2 + 3 + \cdots + n$.

Solution

(i) Given $1 + 2 + 3 + \cdots + n = 120$ i.e. $\frac{n(n+1)}{2} = 120$

$$\therefore \quad 1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2 = 120^2 = 14400$$

(ii) Given $1^3 + 2^3 + 3^3 + \cdots + n^3 = 36100$

$$\Longrightarrow \quad \left(\frac{n(n+1)}{2}\right)^2 = 36100 = 19 \times 19 \times 10 \times 10$$

$$\Longrightarrow \quad \frac{n(n+1)}{2} = 190$$

Thus, $1 + 2 + 3 + \cdots + n = 190$.

Example 2.34

Find the total area of 14 squares whose sides are 11 cm, 12 cm, $\cdots$, 24 cm, respectively.

Solution The areas of the squares form the series $11^2 + 12^2 + \cdots + 24^2$

$$\begin{aligned}
\text{Total area of 14 squares } &= 11^2 + 12^2 + 13^2 + \cdots + 24^2 \\
&= (1^2 + 2^2 + 3^2 + \cdots + 24^2) - (1^2 + 2^2 + 3^2 + \cdots + 10^2) \\
&= \sum_1^{24} n^2 - \sum_1^{10} n^2 \\
&= \frac{24(24+1)(48+1)}{6} - \frac{10(10+1)(20+1)}{6} \\
&= \frac{(24)(25)(49)}{6} - \frac{(10)(11)(21)}{6} \\
&= 4900 - 385 \\
&= 4515 \text{ sq. cm.}
\end{aligned}$$

Points to Remember

- A sequence of real numbers is an arrangement or a list of real numbers in a specific order.
- The sequence given by $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$, $n = 3, 4, \cdots$ is called the Fibonacci sequence which is nothing but $1, 1, 2, 3, 5, 8, 13, 21, 34, \cdots$
- A sequence $a_1, a_2, a_3, \cdots, a_n, \cdots$ is called an arithmetic sequence if $a_{n+1} = a_n + d$, $n \in \mathbb{N}$ where d is a constant. Here a_1 is called the first term and the constant d is called the common difference.

  The formula for the general term of an A.P. is $t_n = a + (n-1)d \quad \forall \; n \in \mathbb{N}$.
- A sequence $a_1, a_2, a_3, \cdots, a_n, \cdots$ is called a geometric sequence if $a_{n+1} = a_n r$, where $r \neq 0$, $n \in \mathbb{N}$ where r is a constant. Here, a_1 is the first term and the constant r is called the common ratio. The formula for the general term of a G.P. is $t_n = ar^{n-1}$, $n = 1, 2, 3, \cdots$.
- An expression of addition of terms of a sequence is called a series. If the sum consists only finite number of terms, then it is called a finite series. If the sum consists of infinite number of terms of a sequence, then it is called an infinite series.
- The sum S_n of the first n terms of an arithmetic sequence with first term a and common difference d is given by $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$, where l is the last term.

- The sum of the first n terms of a geometric series is given by

$$S_n = \begin{cases} \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}, & \text{if } r \neq 1 \\ na & \text{if } r = 1. \end{cases}$$

where a is the first term and r is the common ratio.

- The sum of the first n natural numbers, $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.
- The sum of the first n odd natural numbers, $\sum_{k=1}^{n}(2k-1) = n^2$
- The sum of first n odd natural numbers (when the last term l is given) is

$$1 + 3 + 5 + \cdots + l = \left(\frac{l+1}{2}\right)^2.$$

- The sum of squares of first n natural numbers, $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.
- The sum of cubes of the first n natural numbers, $\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$.

Example 3.1

Solve $3x - 5y = -16$, $2x + 5y = 31$

Solution The given equations are

$$3x - 5y = -16 \qquad (1)$$

$$2x + 5y = 31 \qquad (2)$$

Note that the coefficients of y in both equations are numerically equal.

So, we can eliminate y easily.

Adding (1) and (2), we obtain an equation

$$5x = 15 \qquad (3)$$

That is, $x = 3$.

Now, we substitute $x = 3$ in (1) or (2) to solve for y.

Substituting $x = 3$ in (1) we obtain, $3(3) - 5y = -16$

$$\Longrightarrow \quad y = 5.$$

Now, (3, 5) a is solution to the given system because (1) and (2) are true when $x = 3$ and $y = 5$ as from (1) and (2) we get, $3(3) - 5(5) = -16$ and $2(3) + 5(5) = 31$.

Example 3.2

The cost of 11 pencils and 3 erasers is ₹ 50 and the cost of 8 pencils and 3 erasers is ₹ 38. Find the cost of each pencil and each eraser.

Solution Let x denote the cost of a pencil in rupees and y denote the cost of an eraser in rupees.

Then according to the given information we have

$$11x + 3y = 50 \qquad (1)$$

$$8x + 3y = 38 \qquad (2)$$

Subtracting (2) from (1) we get, $3x = 12$ which gives $x = 4$.

Now substitute $x = 4$ in (1) to find the value of y. We get,

$$11(4) + 3y = 50 \quad \text{i.e.,} \quad y = 2.$$

Therefore, $x = 4$ and $y = 2$ is the solution of the given pair of equations.

Thus, the cost of a pencil is ₹ 4 and that of an eraser is ₹ 2.

Example 3.3

Solve by elimination method $3x + 4y = -25$, $2x - 3y = 6$

Solution The given system is

$$3x + 4y = -25 \qquad (1)$$

$$2x - 3y = 6 \qquad (2)$$

To eliminate the variable x, let us multiply (1) by 2 and (2) by –3 to obtain

$$(1) \times 2 \implies 6x + 8y = -50 \qquad (3)$$

$$(2) \times -3 \implies -6x + 9y = -18 \qquad (4)$$

Now, adding (3) and (4) we get, $17y = -68$ which gives $y = -4$

Next, substitute $y = -4$ in (1) to obtain

$$3x + 4(-4) = -25$$

That is, $x = -3$

Hence, the solution is $(-3, -4)$.

Example 3.16

(i) Prove that $x-1$ is a factor of $x^3-6x^2+11x-6$.

(ii) Prove that $x+1$ is a factor of $x^3+6x^2+11x+6$.

Solution

(i) Let $p(x) = x^3-6x^2+11x-6$.

$p(1) = 1-6+11-6 = 0$. (note that sum of the coefficients is 0)

Thus, $(x-1)$ is a factor of $p(x)$.

(ii) Let $q(x) = x^3+6x^2+11x+6$.

$q(-1) = -1+6-11+6 = 0$. Hence, $x+1$ is a factor of $q(x)$

3.5 Greatest Common Divisor (GCD) and Least Common Multiple (LCM)

3.5.1 Greatest Common Divisor (GCD)

The Highest Common Factor (HCF) or Greatest Common Divisor (GCD) of two or more algebraic expressions is the expression of highest degree which divides each of them without remainder.

Consider the simple expressions

(i) a^4, a^3, a^5, a^6 (ii) a^3b^4, ab^5c^2, a^2b^7c

In (i), note that a, a^2, a^3 are the divisors of all these expressions. Out of them, a^3 is the divisor with highest power. Therefore a^3 is the GCD of the expressions a^4, a^3, a^5, a^6.

In (ii), similarly, one can easily see that ab^4 is the GCD of a^3b^4, ab^5c^2, a^2b^7c.

If the expressions have numerical coefficients, find their greatest common divisor, and prefix it as a coefficient to the greatest common divisor of the algebraic expressions.

Let us consider a few more examples to understand the greatest common divisor.

Examples 3.19

Find the GCD of the following : (i) 90, 150, 225 (ii) $15x^4y^3z^5$, $12x^2y^7z^2$

(iii) $6(2x^2-3x-2)$, $8(4x^2+4x+1)$, $12(2x^2+7x+3)$

Solution

(i) Let us write the numbers 90, 150 and 225 in the product of their prime factors as

$$90 = 2\times3\times3\times5\ ,\ 150 = 2\times3\times5\times5 \text{ and } 225 = 3\times3\times5\times5$$

From the above 3 and 5 are common prime factors of all the given numbers.

Hence the GCD $= 3\times5 = 15$

(ii) We shall use similar technique to find the GCD of algebraic expressions.

Now let us take the given expressions $15x^4y^3z^5$ and $12x^2y^7z^2$.

Here the common divisors of the given expressions are 3, x^2, y^3 and z^2.

Therefore, GCD $= 3\times x^2\times y^3\times z^2 = 3x^2y^3z^2$

(iii) Given expressions are $6(2x^2-3x-2)$, $8(4x^2+4x+1)$, $12(2x^2+7x+3)$

Now, GCD of 6, 8, 12 is 2

Next let us find the factors of quadratic expressions.

$$2x^2-3x-2 = (2x+1)(x-2)$$

$$4x^2+4x+1 = (2x+1)(2x+1)$$

$$2x^2+7x+3 = (2x+1)(x+3)$$

Common factor of the above quadratic expressions is $(2x+1)$.

Therefore, GCD $= 2(2x+1)$.

3.5.2 Greatest common divisor of polynomials using division algorithm

First let us consider the simple case of finding GCD of 924 and 105.

$924 = 8\times105+84$

$105 = 1\times84+21,$ (or)

$84 = 4\times21+0,$

21 is the GCD of 924 and 105

$$\begin{array}{r|r} & 8 \\ \hline 105 & 924 \\ & 840 \\ \hline & 84 \end{array} \quad \begin{array}{r|r} & 1 \\ \hline 84 & 105 \\ & 84 \\ \hline & 21 \end{array} \quad \begin{array}{r|r} & 4 \\ \hline 21 & 84 \\ & 84 \\ \hline & 0 \end{array}$$

Similar technique works with polynomials when they have GCD.

3.5.3 Least Common Multiple (LCM)

The least common multiple of two or more algebraic expressions is the expression of lowest degree which is divisible by each of them without remainder. For example, consider the simple expressions a^4, a^3, a^6.

Now, $a^6,\ a^7,\ a^8, \cdots$ are common multiples of a^3, a^4 and a^6.

Of all the common multiples, the least common multiple is a^6

Hence LCM of a^4, a^3, a^6 is a^6. Similarly, a^3b^7 is the LCM of a^3b^4, ab^5, a^2b^7.

We shall consider some more examples of finding LCM.

Example 3.22

Find the LCM of the following.

(i) 90, 150, 225 (ii) $35a^2c^3b,\ 42a^3cb^2,\ 30ac^2b^3$

(iii) $(a-1)^5(a+3)^2,\ (a-2)^2(a-1)^3(a+3)^4$

(iv) $x^3+y^3,\ x^3-y^3,\ x^4+x^2y^2+y^4$

Solution

(i) Now,
$$90 = 2\times3\times3\times5 = 2^1\times3^2\times5^1$$
$$150 = 2\times3\times5\times5 = 2^1\times3^1\times5^2$$
$$225 = 3\times3\times5\times5 = 3^2\times5^2$$

The product $2^1\times3^2\times5^2 = 450$ is the required LCM.

(ii) Now, LCM of 35, 42 and 30 is $5\times7\times6 = 210$

Hence, the required LCM $= 210\times a^3\times c^3\times b^3 = 210a^3c^3b^3$.

(iii) Now, LCM of $(a-1)^5(a+3)^2,\ (a-2)^2(a-1)^3(a+3)^4$ is $(a-1)^5(a+3)^4(a-2)^2$.

(iv) Let us first find the factors for each of the given expressions.
$$x^3+y^3 = (x+y)(x^2-xy+y^2)$$
$$x^3-y^3 = (x-y)(x^2+xy+y^2)$$
$$x^4+x^2y^2+y^4 = (x^2+y^2)^2-x^2y^2 = (x^2+xy+y^2)(x^2-xy+y^2)$$

Thus,
$$\text{LCM} = (x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$$
$$= (x^3+y^3)(x^3-y^3) = x^6-y^6.$$

Exercise 3.7

Find the LCM of the following.

1. $x^3y^2,\ xyz$
2. $3x^2yz,\ 4x^3y^3$
3. $a^2bc,\ b^2ca,\ c^2ab$
4. $66a^4b^2c^3,\ 44a^3b^4c^2,\ 24a^2b^3c^4$
5. $a^{m+1},\ a^{m+2},\ a^{m+3}$
6. $x^2y+xy^2,\ x^2+xy$
7. $3(a-1),\ 2(a-1)^2,\ (a^2-1)$
8. $2x^2-18y^2,\ 5x^2y+15xy^2,\ x^3+27y^3$
9. $(x+4)^2(x-3)^3,\ (x-1)(x+4)(x-3)^2$
10. $10(9x^2+6xy+y^2),\ 12(3x^2-5xy-2y^2),\ 14(6x^4+2x^3)$.

3.5.4 Relation between LCM and GCD

We know that the product of two positive integers is equal to the product of their LCM and GCD. For example, $21 \times 35 = 105 \times 7$, where LCM (21,35) =105 and GCD (21,35) = 7.

In the same way, we have the following result:

The product of any two polynomials is equal to the product of their LCM and GCD.

That is, $f(x) \times g(x) = \text{LCM}\,(f(x), g(x)) \times \text{GCD}\,(f(x), g(x))$.

Let us justify this result with an example.

Let $f(x) = 12(x^4 - x^3)$ and $g(x) = 8(x^4 - 3x^3 + 2x^2)$ be two polynomials.

Now, $f(x) = 12(x^4 - x^3) = 2^2 \times 3 \times x^3 \times (x-1)$ (1)

Also, $g(x) = 8(x^4 - 3x^3 + 2x^2) = 2^3 \times x^2 \times (x-1) \times (x-2)$ (2)

From (1) and (2) we get,

$$\text{LCM}\,(f(x), g(x)) = 2^3 \times 3^1 \times x^3 \times (x-1) \times (x-2) = 24x^3(x-1)(x-2)$$

$$\text{GCD}\,(f(x), g(x)) = 4x^2(x-1)$$

Therefore, $\text{LCM} \times \text{GCD} = 24x^3(x-1)(x-2) \times 4x^2(x-1)$

$= 96x^5(x-1)^2(x-2)$ (3)

Also, $f(x) \times g(x) = 12x^3(x-1) \times 8x^2(x-1)(x-2)$

$= 96x^5(x-1)^2(x-2)$ (4)

From (3) and (4) we obtain, $\text{LCM} \times \text{GCD} = f(x) \times g(x)$.

Thus, the product of LCM and GCD of two polynomials is equal to the product of the two polynomials. Further, if $f(x)$, $g(x)$ and one of LCM and GCD are given, then the other can be found without ambiguity because LCM and GCD are unique, except for a factor of -1.

Example 3.23

The GCD of $x^4 + 3x^3 + 5x^2 + 26x + 56$ and $x^4 + 2x^3 - 4x^2 - x + 28$ is $x^2 + 5x + 7$. Find their LCM.

Solution Let $f(x) = x^4 + 3x^3 + 5x^2 + 26x + 56$ and $g(x) = x^4 + 2x^3 - 4x^2 - x + 28$

Given that $\text{GCD} = x^2 + 5x + 7$. Also, we have $\text{GCD} \times \text{LCM} = f(x) \times g(x)$.

Thus, $\text{LCM} = \dfrac{f(x) \times g(x)}{\text{GCD}}$ (1)

Now, GCD divides both $f(x)$ and $g(x)$.

Let us divide $f(x)$ by the GCD.

$$\begin{array}{r|rrrrr}
 & 1 & -2 & 8 & & \\
\hline
1\ \ 5\ \ 7 & 1 & 3 & 5 & 26 & 56 \\
 & 1 & 5 & 7 & & \\
\hline
 & & -2 & -2 & 26 & \\
 & & -2 & -10 & -14 & \\
\hline
 & & & 8 & 40 & 56 \\
 & & & 8 & 40 & 56 \\
\hline
 & & & & 0 &
\end{array}$$

When $f(x)$ is divided by GCD, we get the quotient as $x^2 - 2x + 8$.

Now, $(1) \Longrightarrow$ LCM $= (x^2 - 2x + 8) \times g(x)$

Thus, LCM $= (x^2 - 2x + 8)(x^4 + 2x^3 - 4x^2 - x + 28)$.

Note

In the above problem, we can also divide $g(x)$ by GCD and multiply the quotient by $f(x)$ to get the required LCM.

Example 3.24

The GCD and LCM of two polynomials are $x + 1$ and $x^6 - 1$ respectively. If one of the polynomials is $x^3 + 1$, find the other.

Solution Given GCD $= x + 1$ and LCM $= x^6 - 1$

Let $f(x) = x^3 + 1$.

We know that LCM $\times$ GCD $= f(x) \times g(x)$

$$\Longrightarrow g(x) = \frac{\text{LCM} \times \text{GCD}}{f(x)} = \frac{(x^6 - 1)(x + 1)}{x^3 + 1}$$

$$= \frac{(x^3 + 1)(x^3 - 1)(x + 1)}{x^3 + 1} = (x^3 - 1)(x + 1)$$

Hence, $g(x) = (x^3 - 1)(x + 1)$.

Example 3.25

Simplify the rational expressions into lowest forms.

(i) $\frac{5x+20}{7x+28}$ (ii) $\frac{x^3-5x^2}{3x^3+2x^4}$

(iii) $\frac{6x^2-5x+1}{9x^2+12x-5}$ (iv) $\frac{(x-3)(x^2-5x+4)}{(x-1)(x^2-2x-3)}$

Solution

(i) Now, $\frac{5x+20}{7x+28} = \frac{5(x+4)}{7(x+4)} = \frac{5}{7}$

(ii) Now, $\frac{x^3-5x^2}{3x^3+2x^4} = \frac{x^2(x-5)}{x^3(2x+3)} = \frac{x-5}{x(2x+3)}$

(iii) Let $p(x)= 6x^2-5x+1= (2x-1)(3x-1)$ and

$q(x) = 9x^2+12x-5= (3x+5)(3x-1)$

Therefore, $\frac{p(x)}{q(x)} = \frac{(2x-1)(3x-1)}{(3x+5)(3x-1)} = \frac{2x-1}{3x+5}$

(iv) Let $f(x) = (x-3)(x^2-5x+4)= (x-3)(x-1)(x-4)$ and

$g(x) = (x-1)(x^2-2x-3)= (x-1)(x-3)(x+1)$

Therefore, $\frac{f(x)}{g(x)} = \frac{(x-3)(x-1)(x-4)}{(x-1)(x-3)(x+1)} = \frac{x-4}{x+1}$

Example 3.28

Simplify (i) $\frac{x+2}{x+3}+\frac{x-1}{x-2}$ (ii) $\frac{x+1}{(x-1)^2}+\frac{1}{x+1}$ (iii) $\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12}$

Solution

(i) $\frac{x+2}{x+3}+\frac{x-1}{x-2} = \frac{(x+2)(x-2)+(x-1)(x+3)}{(x+3)(x-2)} = \frac{2x^2+2x-7}{x^2+x-6}$

(ii) $\frac{x+1}{(x-1)^2}+\frac{1}{x+1} = \frac{(x+1)^2+(x-1)^2}{(x-1)^2(x+1)} = \frac{2x^2+2}{(x-1)^2(x+1)}$

$= \frac{2x^2+2}{x^3-x^2-x+1}$

(iii) $\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12} = \frac{(x-3)(x+2)}{(x+3)(x-3)}+\frac{(x+6)(x-4)}{(x+3)(x-4)}$

$= \frac{x+2}{x+3}+\frac{x+6}{x+3} = \frac{x+2+x+6}{x+3} = \frac{2x+8}{x+3}$

3.7.1 Square root by factorization method

Example 3.31

Find the square root of

(i) $121(x-a)^4(x-b)^6(x-c)^{12}$ (ii) $\frac{81x^4y^6z^8}{64w^{12}s^{14}}$ (iii) $(2x+3y)^2-24xy$

Solution

(i) $\sqrt{121(x-a)^4(x-b)^6(x-c)^{12}} = 11\left|(x-a)^2(x-b)^3(x-c)^6\right|$

(ii) $\sqrt{\frac{81x^4y^6z^8}{64w^{12}s^{14}}} = \frac{9}{8}\left|\frac{x^2y^3z^4}{w^6s^7}\right|$

(iii) $\sqrt{(2x+3y)^2-24xy} = \sqrt{4x^2+12xy+9y^2-24xy} = \sqrt{(2x-3y)^2}$

Example 3.42

The sum of a number and its reciprocal is $5\frac{1}{5}$. Find the number.

Solution Let x denote the required number. Then its reciprocal is $\frac{1}{x}$

By the given condition, $x+\frac{1}{x} = 5\frac{1}{5} \implies \frac{x^2+1}{x} = \frac{26}{5}$

So, $5x^2-26x+5 = 0$

$\implies 5x^2-25x-x+5 = 0$

That is, $(5x-1)(x-5) = 0 \implies x = 5$ or $\frac{1}{5}$

Thus, the required numbers are $5, \frac{1}{5}$.

Example 3.43

The base of a triangle is 4cm longer than its altitude. If the area of the triangle is 48 sq. cm, then find its base and altitude.

Solution Let the altitude of the triangle be x cm.

By the given condition, the base of the triangle is $(x+4)$ cm.

Now, the area of the triangle = $\frac{1}{2}(\text{base})\times\text{height}$

By the given condition $\frac{1}{2}(x+4)(x) = 48$

$\implies x^2+4x-96 = 0 \implies (x+12)(x-8) = 0$

$\implies x = -12$ or 8

But $x=-12$ is not possible (since the length should be positive)

Therefore, $x=8$ and hence, $x+4=12$.

Thus, the altitude of the triangle is 8cm and the base of the triangle is 12cm.

Example 3.44

A car left 30 minutes later than the scheduled time. In order to reach its destination 150km away in time, it has to increase its speed by 25km/hr from its usual speed. Find its usual speed.

Solution Let the usual speed of the car be x km/hr.

Thus, the increased speed of the car is $(x+25)$ km/hr

Total distance $=150$ km; Time taken $=\dfrac{\text{Distance}}{\text{Speed}}$.

Let T_1 and T_2 be the time taken in hours by the car to cover the given distance in scheduled time and decreased time (as the speed is increased) respectively.

By the given information $T_1 - T_2 = \frac{1}{2}$hr (30 minutes $= \frac{1}{2}$hr)

$$\Longrightarrow \frac{150}{x} - \frac{150}{x+25} = \frac{1}{2} \Longrightarrow 150\left[\frac{x+25-x}{x(x+25)}\right] = \frac{1}{2}$$

$$\Longrightarrow x^2 + 25x - 7500 = 0 \Longrightarrow (x+100)(x-75) = 0$$

Thus, $x = 75$ or -100, but $x = -100$ is not an admissible value.

Therefore, the usual speed of the car is 75 km/hr.

Note

If α and β are the roots of $ax^2+bx+c=0$, then many expressions in α and β like $\alpha^2+\beta^2$, $\alpha^2\beta^2$, $\alpha^2-\beta^2$ etc., can be evaluated using the values of $\alpha+\beta$ and $\alpha\beta$.

Let us write some results involving α and β.

(i) $|\alpha-\beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

(ii) $\alpha^2+\beta^2 = [(\alpha+\beta)^2 - 2\alpha\beta]$

(iii) $\alpha^2-\beta^2 = (\alpha+\beta)(\alpha-\beta) = (\alpha+\beta)[\sqrt{(\alpha+\beta)^2-4\alpha\beta}]$ only if $\alpha \geq \beta$

(iv) $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

(v) $\alpha^3-\beta^3 = (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta)$

(vi) $\alpha^4+\beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2 = [(\alpha+\beta)^2-2\alpha\beta]^2 - 2(\alpha\beta)^2$

(vii) $\alpha^4-\beta^4 = (\alpha+\beta)(\alpha-\beta)(\alpha^2+\beta^2)$

8.2.1 Right Circular Cylinder

If we take a number of circular sheets of paper or cardboard of the same shape and size and stack them up in a vertical pile, then by this process, we shall obtain a solid object known as a Right Circular Cylinder. Note that it has been kept at right angles to the base, and the base is circular. (See Fig. 8.3)

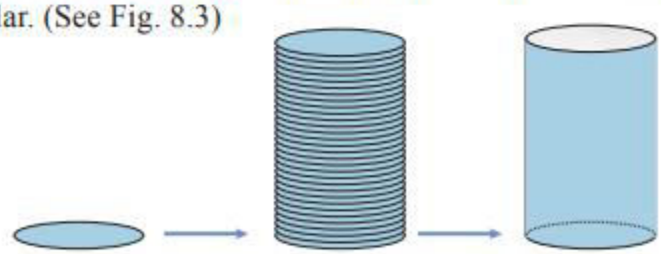


Fig. 8.3

Definition

If a rectangle revolves about its one side and completes a full rotation, the solid thus formed is called a right circular cylinder.

Activity

Let $ABCD$ be a rectangle. Assume that it revolves about its side AB and completes a full rotation. This revolution generates a right circular cylinder as shown in the figures. AB is called the axis of the cylinder. The length AB is the length or the height of the cylinder and AD or BC is called its radius.

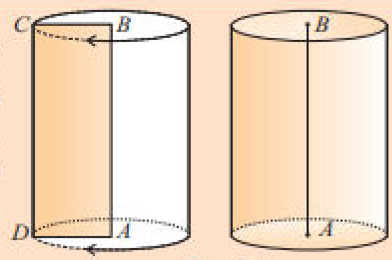


Fig. 8.4

Note

(i) If the base of a cylinder is not circular then it is called oblique cylinder.

(ii) If the base is circular but not perpendicular to the axis of the cylinder, then the cylinder is called circular cylinder.

(iii) If the axis is perpendicular to the circular base, then the cylinder is called right circular cylinder.

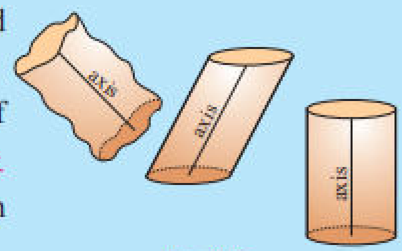


Fig. 8.5

(i) Curved Surface area of a solid right circular cylinder

In the adjoining figure, the bottom and top face of the right circular cylinder are concurrent circular regions, parallel to each other. The vertical surface of the cylinder is curved and hence its area is called the curved surface or lateral surface area of the cylinder.

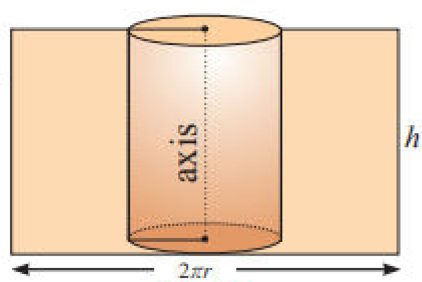


Fig. 8.6

Curved Surface Area of a cylinder, CSA = Circumference of the base $\times$ Height $= 2\pi r \times h$

$$= 2\pi rh \text{ sq. units.}$$

(ii) Total Surface Area of a solid right circular cylinder

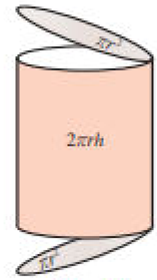


Fig. 8.7

Total Surface Area, TSA = Area of the Curved Surface Area + 2 × Base Area

$= 2\pi rh + 2 \times \pi r^2$

Thus, TSA $= 2\pi r(h + r)$ sq.units.

(iii) Right circular hollow cylinder

Solids like iron pipe, rubber tube, etc., are in the shape of hollow cylinders. For a hollow cylinder of height h with external and internal radii R and r respectively,

we have, curved surface area, CSA = External surface area + Internal surface area

$= 2\pi Rh + 2\pi rh$

Thus, CSA $= 2\pi h(R + r)$ sq.units

Total surface area, TSA = CSA + 2 × Base area

$= 2\pi h(R + r) + 2 \times [\pi R^2 - \pi r^2]$

$= 2\pi h(R + r) + 2\pi(R + r)(R - r)$

$\therefore$ TSA $= 2\pi(R + r)(R - r + h)$ sq.units.

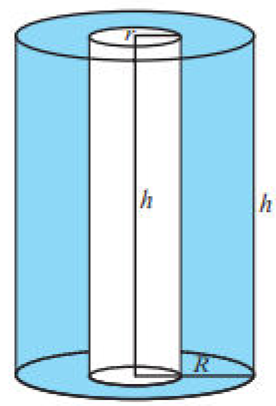


Fig. 8.8

Remark

Thickness of the hollow cylinder, w = $R - r$.

Note

In this chapter, for π we take an approximate value $\frac{22}{7}$ whenever it is required.

Example 8.1

A solid right circular cylinder has radius 7cm and height 20cm. Find its (i) curved surface area and (ii) total surface area. (Take $\pi = \frac{22}{7}$)

Solution Let r and h be the radius and height of the solid right circular cylinder respectively.

Given that $r = 7$cm and $h = 20$cm

Curved surface area, CSA $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 20$

Thus, the curved surface area = 880 sq.cm

Now, the total surface area $= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 7 \times [20 + 7] = 44 \times 27$

Thus, the total surface area = 1188 sq.cm.

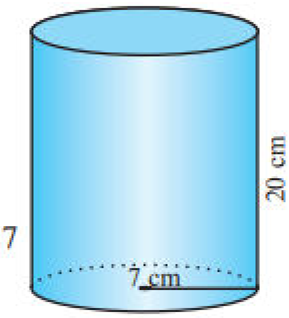


Fig. 8.9

Example 8.2

If the total surface area of a solid right circular cylinder is 880 sq.cm and its radius is 10 cm, find its curved surface area. (Take $\pi = \frac{22}{7}$)

Solution Let r and h be the radius and height of the solid right circular cylinder respectively.

Let S be the total surface area of the solid right circular cylinder.

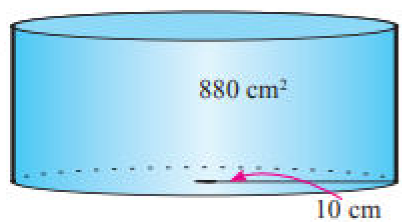


Fig. 8.10

Given that $r = 10$ cm and $S = 880$ cm^2

Now, $S = 880 \implies 2\pi r[h + r] = 880$

$\implies 2 \times \frac{22}{7} \times 10[h + 10] = 880$

$\implies h + 10 = \frac{880 \times 7}{2 \times 22 \times 10}$

$\implies h + 10 = 14$

Thus, the height of the cylinder, $h = 4$cm

Now, the curved surface area, CSA is

$2\pi rh = 2 \times \frac{22}{7} \times 10 \times 4 = \frac{1760}{7}$

Aliter :

CSA = TSA – 2× Area of the base

$= 880 - 2 \times \pi r^2$

$= 880 - 2 \times \frac{22}{7} \times 10^2$

$= \frac{1760}{7} = 251\frac{3}{7}$ sq.cm.

Thus, the curved surface area of the cylinder = $251\frac{3}{7}$ sq.cm.

Example 8.3

The ratio between the base radius and the height of a solid right circular cylinder is 2 : 5. If its curved surface area is $\frac{3960}{7}$ sq.cm, find the height and radius. (use $\pi = \frac{22}{7}$)

Solution Let r and h be the radius and height of the right circular cylinder respectively.

Given that $r : h = 2 : 5 \implies \frac{r}{h} = \frac{2}{5}$. Thus, $r = \frac{2}{5}h$

Now, the curved surface area, CSA = $2\pi rh$

$$\implies \quad 2\times\frac{22}{7}\times\frac{2}{5}\times h\times h = \frac{3960}{7}$$

$$\implies \quad h^2 = \frac{3960\times 7\times 5}{2\times 22\times 2\times 7} = 225$$

Thus, $h = 15 \implies r = \frac{2}{5}h = 6.$

Hence, the height of the cylinder is 15 cm and the radius is 6 cm.

Example 8.5

The internal and external radii of a hollow cylinder are 12 cm and 18 cm respectively. If its height is 14cm, then find its curved surface area and total surface area. (Take $\pi = \frac{22}{7}$)

Solution Let r, R and h be the internal and external radii and the height of a hollow cylinder respectively.

Given that $r = 12$ cm, $R = 18$ cm, $h = 14$ cm

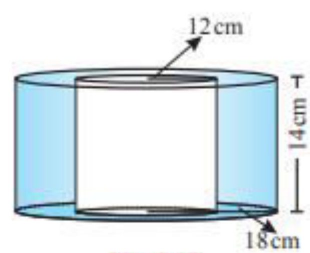


Fig. 8.12

Now, curved surface area, CSA = $2\pi h(R+r)$

Thus, CSA = $2\times\frac{22}{7}\times 14\times(18+12)$

= 2640 sq.cm

Total surface area, TSA $= 2\pi(R+r)(R-r+h)$

$$= 2\times\frac{22}{7}\times(18+12)(18-12+14)$$

$$= 2\times\frac{22}{7}\times 30\times 20 = \frac{26400}{7}.$$

Thus, the total surface area = $3771\frac{3}{7}$ sq.cm.

Fig. 8.13

The length AB is called the height of the cone.

The length BC is called the radius of its base ($BC = r$). The length AC is called the slant height l of the cone ($AC = AD = l$).

In the right angled $\triangle ABC$

We have, $l = \sqrt{h^2 + r^2}$ (Pythagoras theorem)

$h = \sqrt{l^2 - r^2}$

$r = \sqrt{l^2 - h^2}$

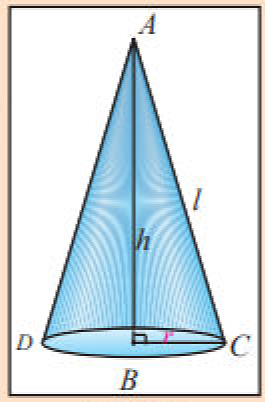


Fig. 8.14

(i) Curved surface area of a hollow cone

Let us consider a sector with radius l and central angle θ°. Let L denote the length of the arc. Thus, $\frac{2\pi l}{L} = \frac{360^\circ}{\theta^\circ}$

$\Longrightarrow \quad L = 2\pi l \times \frac{\theta^\circ}{360^\circ} \qquad (1)$

Now, join the radii of the sector to obtain a right circular cone.

Let r be the radius of the cone.

Hence, $L = 2\pi r$

From (1) we obtain,

$2\pi r = 2\pi l \times \frac{\theta^\circ}{360^\circ}$

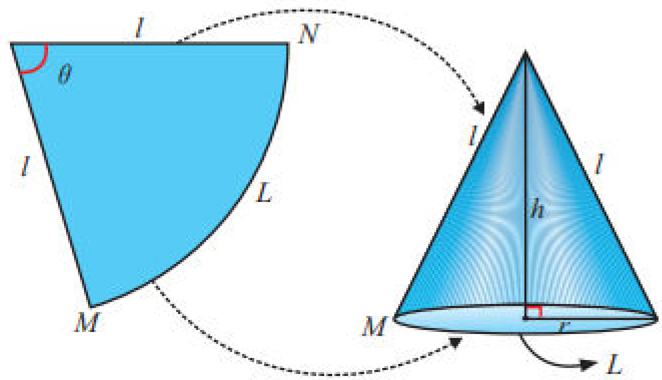


Fig. 8.16

$$\Longrightarrow \quad r = l\left(\frac{\theta^\circ}{360^\circ}\right)$$

$$\Longrightarrow \quad \frac{r}{l} = \left(\frac{\theta^\circ}{360^\circ}\right)$$

Let A be the area of the sector. Then

$$\frac{\pi l^2}{A} = \frac{360^\circ}{\theta^\circ} \qquad (2)$$

Remarks

When a sector of a circle is folded into a cone, the following conversions are taking place:

Sector	Cone
Radius (l)	→ Slant height (l)
Arc Length (L)	→ Perimeter of the base $2\pi r$
Area	→ Curved Surface Area πrl

Then the curved surface area of the cone $\Big\}$ = Area of the sector

Thus, the area of the curved surface of the cone $\Big\}$ $A = \pi l^2\left(\frac{\theta^\circ}{360^\circ}\right) = \pi l^2\left(\frac{r}{l}\right)$.

Hence, the curved surface area of the cone = πrl sq.units.

(ii) Total surface area of the solid right circular cone

Total surface area of the solid cone = $\Big\{$ Curved surface area of the cone + Area of the base

$$= \pi rl + \pi r^2$$

Total surface area of the solid cone = $\pi r(l + r)$ sq.units.

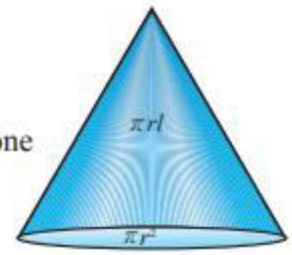


Fig. 8.17

Example 8.6

Radius and slant height of a solid right circular cone are 35cm and 37cm respectively. Find the curved surface area and total surface area of the cone. (Take $\pi = \frac{22}{7}$)

Solution Let r and l be the radius and the slant height of the solid right circular c respectively.

$$r = 35 \text{ cm}, \ l = 37 \text{ cm}$$

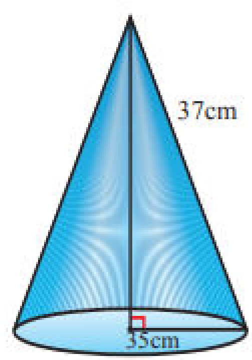
Curved surface area, CSA $= \pi rl = \pi(35)(37)$

CSA = 4070 sq.cm

Total surface area, TSA $= \pi r[l + r]$

$$= \frac{22}{7} \times 35 \times [37 + 35]$$

Thus, TSA = 7920 sq.cm.



Example 8.7

Let O and C be the centre of the base and the vertex of a right circular cone. Let B be any point on the circumference of the base. If the radius of the cone is 6 cm and if $\angle OBC = 60^\circ$, then find the height and curved surface area of the cone.

Solution Given that radius OB = 6 cm and $\angle OBC = 60^\circ$.

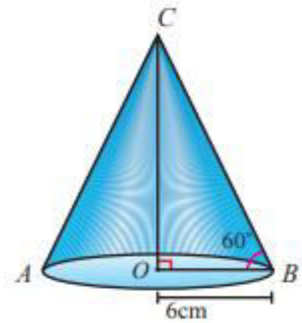


Fig. 8.19

In the right angled $\triangle OBC$,

$$\cos 60^\circ = \frac{OB}{BC}$$

$$\Longrightarrow \quad BC = \frac{OB}{\cos 60^\circ}$$

$$\therefore \quad BC = \frac{6}{\left(\frac{1}{2}\right)} = 12 \text{ cm}$$

Thus, the slant height of the cone, $l = 12$ cm

In the right angled $\triangle OBC$, we have

$$\tan 60^\circ = \frac{OC}{OB}$$

$$\Longrightarrow \quad OC = OB \tan 60^\circ = 6\sqrt{3}$$

Thus, the height of the cone, OC = $6\sqrt{3}$ cm

Now, the curved surface area is $\pi r l = \pi \times 6 \times 12 = 72\pi \text{ cm}^2$.

Example 8.8

A sector containing an angle of 120° is cut off from a circle of radius 21 cm and folded into a cone. Find the curved surface area of the cone. (Take $\pi = \frac{22}{7}$)

Solution Let r be the base radius of the cone.

Angle of the sector, $\theta = 120^\circ$

Radius of the sector, R = 21 cm

When the sector is folded into a right circular cone, we have circumference of the base of the cone

= Length of the arc

$$\implies \quad 2\pi r = \frac{\theta}{360^\circ} \times 2\pi R$$

$$\implies \quad r = \frac{\theta}{360^\circ} \times R$$

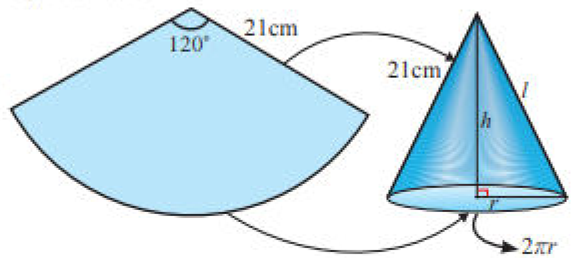


Fig. 8.20

Thus, the base radius of the cone, $r = \frac{120^\circ}{360^\circ} \times 21 = 7$ cm.

Also, the slant height of the cone ,

l = Radius of the sector

Thus, $l = R \implies l = 21$ cm.

Now , the curved surface area of the cone,

CSA $= \pi r l$

$= \frac{22}{7} \times 7 \times 21 = 462.$

Thus, the curved surface area of the cone is 462 sq.cm.

Aliter :

CSA of the cone = Area of the sector

$= \frac{\theta^\circ}{360^\circ} \times \pi \times R^2$

$= \frac{120}{360} \times \frac{22}{7} \times 21 \times 21$

$=$ 462 sq.cm.

8.2.3 Sphere

If a circular disc is rotated about one of its diameter, the solid thus generated is called sphere. Thus sphere is a 3- dimensional object which has surface area and volume.

(i) Curved surface area of a solid sphere

Activity

Take a circular disc, paste a string along a diameter of the disc and rotate it 360°. The object so created looks like a ball. The new solid is called sphere.

The following activity may help us to visualise the surface area of a sphere as four times the area of the circle with the same radius.

- Take a plastic ball.
- Fix a pin at the top of the ball.
- Wind a uniform thread over the ball so as to cover the whole curved surface area.
- Unwind the thread and measure the length of the thread used.
- Cut the thread into four equal parts.
- Place the strings as shown in the figures.

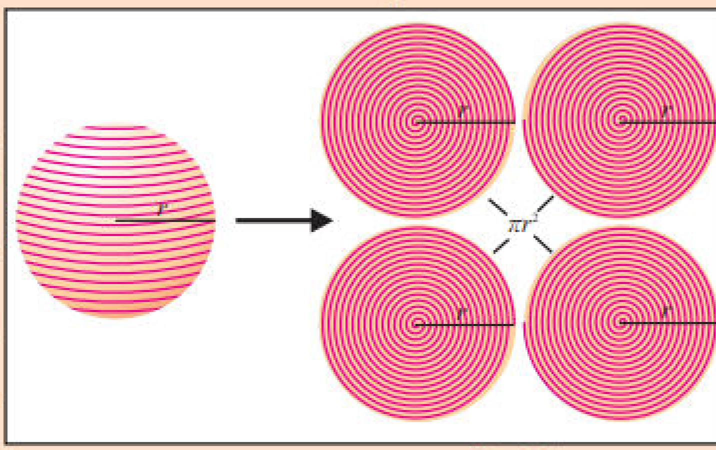


Fig. 8.21

- Measure the radius of the sphere and the circles formed.

Now, the radius of the sphere = radius of the four equal circles.

Thus, curved surface area of the sphere, CSA = $4 \times$ Area of the circle $= 4 \times \pi r^2$

$\therefore$ The curved surface area of a sphere = $4\pi r^2$ sq. units.

(ii) Solid hemisphere

A plane passing through the centre of a solid sphere divides the sphere into two equal parts. Each part of the sphere is called a solid hemisphere.

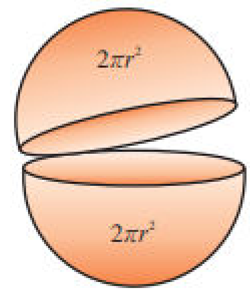


Fig. 8.22

$$\text{Curved surface area of a hemisphere} = \frac{\text{CSA of the Sphere}}{2}$$

$$= \frac{4\pi r^2}{2} = 2\pi r^2 \text{ sq.units.}$$

Total surface area of a hemisphere, TSA = Curved Surface Area + Area of the base Circle

$$= 2\pi r^2 + \pi r^2$$

$$= 3\pi r^2 \text{ sq.units.}$$

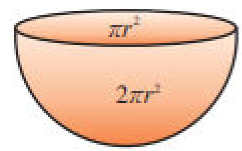


Fig. 8.23

(iii) Hollow hemisphere

Let R and r be the outer and inner radii of the hollow hemisphere.

Now, its curved surface area = Outer surface area + Inner surface area

$$= 2\pi R^2 + 2\pi r^2$$

$$= 2\pi(R^2 + r^2) \text{ sq.units.}$$

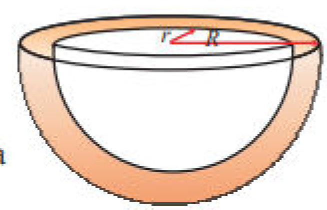


Fig. 8.24

$$\text{The total surface area} = \begin{cases} \text{Outer surface area + Inner surface area} \\ + \text{ Area at the base} \end{cases}$$

$$= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$$

$$= 2\pi(R^2 + r^2) + \pi(R + r)(R - r) \text{ sq.units.}$$

$$= \pi(3R^2 + r^2) \text{ sq. units}$$

Example 8.9

A hollow sphere in which a circus motorcyclist performs his stunts, has an inner diameter of 7 m. Find the area available to the motorcyclist for riding. (Take $\pi = \frac{22}{7}$)

Solution Inner diameter of the hollow sphere, $2r = 7$ m.

Available area to the motorcyclist for riding $=$ Inner surface area of the sphere

$$= 4\pi r^2 = \pi(2r)^2$$

$$= \frac{22}{7} \times 7^2$$

Available area to the motorcyclist for riding $= 154$ sq.m.

Example 8.10

Total surface area of a solid hemisphere is 675π sq.cm. Find the curved surface area of the solid hemisphere.

Solution Given that the total surface area of the solid hemisphere,

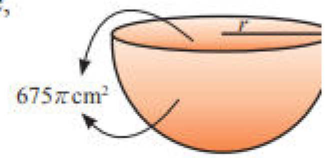


Fig. 8.25

$$3\pi r^2 = 675\pi \text{ sq. cm}$$

$$\Longrightarrow \quad r^2 = 225$$

Now, the curved surface area of the solid hemisphere,

$$\text{CSA} = 2\pi r^2 = 2\pi \times 225 = 450\pi \text{ sq.cm.}$$

Example 8.11

The thickness of a hemispherical bowl is 0.25 cm. The inner radius of the bowl is 5cm. Find the outer curved surface area of the bowl.(Take $\pi = \frac{22}{7}$)

Solution Let r, R and w be the inner and outer radii and thickness of the hemispherical bowl respectively.

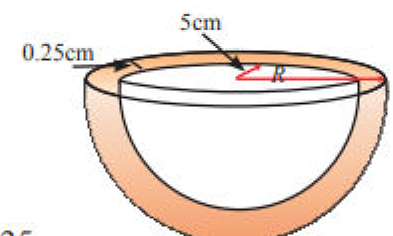


Fig. 8.26

Given that $r = 5\text{cm}, w = 0.25\text{cm}$

$\therefore \quad R = r + w = 5 + 0.25 = 5.25\text{cm}$

Now, outer surface area of the bowl $= 2\pi R^2$

$$= 2 \times \frac{22}{7} \times 5.25 \times 5.25$$

Thus, the outer surface area of the bowl $= 173.25$ sq.cm.

8.3 Volume

So far we have seen the problems related to the surface area of some solids. Now we shall learn how to calculate volumes of some familiar solids. Volume is literally the 'amount of space filled'. The volume of a solid is a numerical characteristic of the solid.

For example, if a body can be decomposed into finite set of unit cubes (cubes of unit sides), then the volume is equal to the number of these cubes.

The cube in the figure, has a volume

$= \text{length} \times \text{width} \times \text{height}$

$= 1\text{ cm} \times 1\text{ cm} \times 1\text{ cm} = 1\text{ cm}^3.$

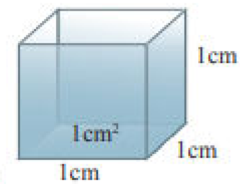


Fig. 8.27

If we say that the volume of an object is 100 cu.cm, then it implies that we need 100 cubes each of 1 cm^3 volume to fill this object completely.

Just like surface area, volume is a positive quantity and is invariant with respect to displacement. Volumes of some solids are illustrated below.

8.3.1 Volume of a right circular cylinder

(i) Volume of a solid right circular cylinder

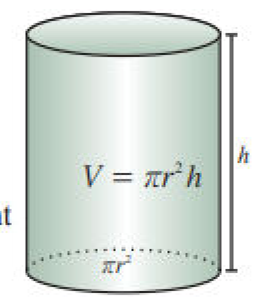


Fig. 8.28

The volume of a solid right circular cylinder is the product of the base area and height.

That is, the volume of the cylinder, $V = \text{Area of the base} \times \text{height}$

$= \pi r^2 \times h$

Thus, the volume of a cylinder, $V = \pi r^2 h$ cu. units.

(ii) Volume of a hollow cylinder (Volume of the material used)

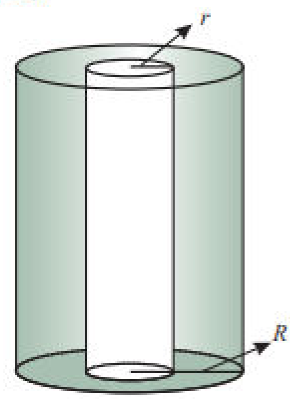


Fig. 8.29

Let R and r be the external and internal radii of a hollow right circular cylinder respectively. Let h be its height.

Then, the volume, $V = \left\{\begin{matrix}\text{Volume of the} \\ \text{outer cylinder}\end{matrix}\right\} - \left\{\begin{matrix}\text{Volume of the} \\ \text{inner cylinder}\end{matrix}\right\}$

$= \pi R^2 h - \pi r^2 h$

Hence, the volume of a hollow cylinder,

$V = \pi h(R^2 - r^2)$ cu. units.

Example 8.12

If the curved surface area of a right circular cylinder is 704 sq.cm, and height is 8 cm find the volume of the cylinder in litres. (Take $\pi = \frac{22}{7}$)

Solution Let r and h be the radius and height of the right circular cylinder respectively.

Given that $h = 8$ cm and CSA = 704 sq.cm

Now, CSA = 704

$\Longrightarrow \quad 2\pi rh = 704$

$2 \times \frac{22}{7} \times r \times 8 = 704$

$\therefore \quad r = \frac{704 \times 7}{2 \times 22 \times 8} = 14$ cm

Fig. 8.30

Thus, the volume of the cylinder, $V = \pi r^2 h$

$= \frac{22}{7} \times 14 \times 14 \times 8$

$= 4928$ cu.cm.

Hence, the volume of the cylinder = 4.928 litres. (1000 cu.cm = 1 litre)

Example 8.13

A hollow cylindrical iron pipe is of length 28 cm. Its outer and inner diameters are 8 cm and 6 cm respectively. Find the volume of the pipe and weight of the pipe if 1 cu.cm of iron weighs 7 gm.(Take $\pi = \frac{22}{7}$)

Solution Let r, R and h be the inner, outer radii and height of the hollow cylindrical pipe respectively.

Given that $2r = 6$ cm, $2R = 8$ cm , $h = 28$ cm

Now, the volume of the pipe, $V = \pi \times h \times (R + r)(R - r)$

$= \frac{22}{7} \times 28 \times (4 + 3)(4 - 3)$

$\therefore$ Volume, $V = 616$ cu. cm

Weight of 1 cu.cm of the metal = 7 gm

Weight of the 616 cu.cm of metal = 7×616 gm

Thus, the weight of the pipe = 4.312 kg.

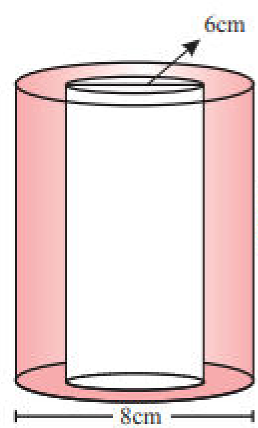


Fig. 8.31

Example 8.14

Base area and volume of a solid right circular cylinder are 13.86 sq.cm, and 69.3 cu.cm respectively. Find its height and curved surface area.(Take $\pi = \frac{22}{7}$)

Solution Let A and V be the base area and volume of the solid right circular cylinder respectively.

Given that the base area, $A = \pi r^2 = 13.86$ sq.cm and

volume, $V = \pi r^2 h = 69.3$ cu.cm.

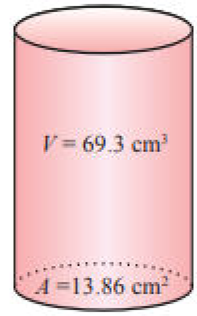


Fig. 8.32

Thus, $\pi r^2 h = 69.3$

$\Longrightarrow \quad 13.86 \times h = 69.3$

$\therefore \quad h = \dfrac{69.3}{13.86} = 5$ cm.

Now, the base area $= \pi r^2 = 13.86$

$$\frac{22}{7} \times r^2 = 13.86$$

$$r^2 = 13.86 \times \frac{7}{22} = 4.41 \quad \Longrightarrow \quad r = \sqrt{4.41} = 2.1 \text{ cm.}$$

Now, Curved surface area, CSA $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 2.1 \times 5$$

Thus, CSA $= 66$ sq.cm.

Example 8.15

The volume of a solid right circular cone is 4928 cu. cm. If its height is 24 cm, then find the radius of the cone. (Take $\pi = \dfrac{22}{7}$)

Solution Let r, h and V be the radius, height and volume of a solid cone respectively.

Given that $V = 4928$ cu.cm and $h = 24$ cm

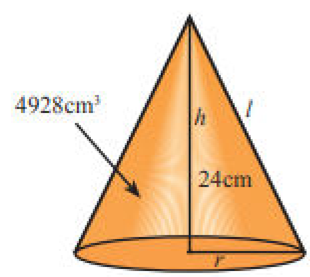


Fig. 8.34

Thus, we have $\dfrac{1}{3}\pi r^2 h = 4928$

$$\Longrightarrow \quad \frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 4928$$

$$\Longrightarrow \quad r^2 = \frac{4928 \times 3 \times 7}{22 \times 24} = 196.$$

Thus, the base radius of the cone, $r = \sqrt{196} = 14$ cm.

Note

* Curved surface area of a frustum of a cone $= \pi(R + r)l$, where $l = \sqrt{h^2 + (R - r)^2}$

* Total surface area of a frustum of a the cone $= \pi l(R + r) + \pi R^2 + \pi r^2, l = \sqrt{h^2 + (R - r)^2}$

(* Not to be used for examination purpose)

Example 8.16

The radii of two circular ends of a frustum shaped bucket are 15 cm and 8 cm. If its depth is 63 cm, find the capacity of the bucket in litres. (Take $\pi = \frac{22}{7}$)

Solution Let R and r are the radii of the circular ends at the top and bottom and h be the depth of the bucket respectively.

Fig. 8.37

Given that $R = 15$ cm , $r = 8$ cm and $h = 63$ cm.

The volume of the bucket (frustum)

$$= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 63 \times (15^2 + 8^2 + 15 \times 8)$$

$$= 26994 \text{ cu.cm.}$$

$$= \frac{26994}{1000} \text{ litres} \qquad (1000 \text{ cu.cm} = 1 \text{ litre})$$

Thus, the capacity of the bucket = 26.994 litres.

8.3.4 Volume of a Sphere

(i) Volume of a Solid Sphere

The following simple experiment justifies the formula for volume of a sphere,

$$V = \frac{4}{3}\pi r^3 \text{ cu.units.}$$

Activity

Take a cylindrical shaped container of radius R and height H. Fill the container with water. Immerse a solid sphere of radius r, where $R > r$, in the container and fill the displaced water into another cylindrical shaped container of radius r and height H. The height of the water level is equal to $\frac{4}{3}$ times of its radius ($h = \frac{4}{3}r$). Now, the volume of the solid sphere is same as that of the displaced water.

Volume of the displaced water, V = Base area x Height

$= \pi r^2 \times \frac{4}{3}r$ (here, height of the water level $h = \frac{4}{3}r$)

$= \frac{4}{3}\pi r^3$

Thus, the volume of the sphere, $V = \frac{4}{3}\pi r^3$ cu.units.

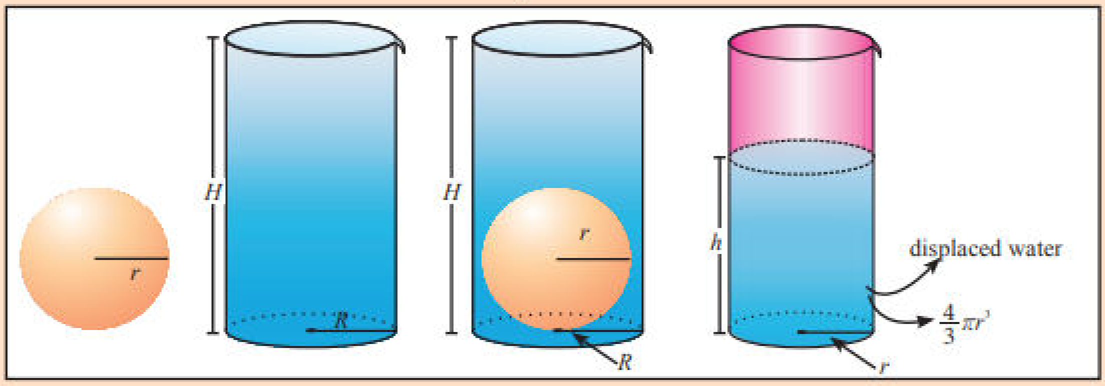


Fig. 8.38

(ii) Volume of a hollow sphere (Volume of the material used)

If the inner and outer radius of a hollow sphere are r and R respectively, then

$$\left.\begin{array}{l}\text{Volume of the}\\ \text{hollow sphere}\end{array}\right\} = \left.\begin{array}{l}\text{Volume of the}\\ \text{outer sphere}\end{array}\right\} - \left\{\begin{array}{l}\text{Volume of the}\\ \text{inner sphere}\end{array}\right.$$

$$= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$

$\therefore$ Volume of hollow sphere $= \frac{4}{3}\pi(R^3 - r^3)$ cu. units.

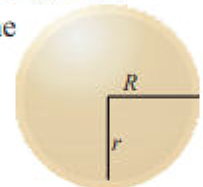


Fig. 8.39

(iii) Volume of a solid hemisphere

Volume of the solid hemisphere $= \frac{1}{2} \times$ volume of the sphere

$$= \frac{1}{2} \times \frac{4}{3}\pi r^3$$

$$= \frac{2}{3}\pi r^3 \text{ cu.units.}$$

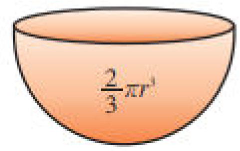


Fig. 8.40

(iv) Volume of a hollow hemisphere (Volume of the material used)

$$\left.\begin{array}{l}\text{Volume of a hollow}\\ \text{hemisphere}\end{array}\right\} = \left.\begin{array}{l}\text{Volume of outer}\\ \text{hemisphere}\end{array}\right\} - \left\{\begin{array}{l}\text{Volume of inner}\\ \text{hemisphere}\end{array}\right.$$

$$= \frac{2}{3} \times \pi \times R^3 - \frac{2}{3} \times \pi \times r^3$$

$$= \frac{2}{3}\pi(R^3 - r^3) \text{ cu.units.}$$

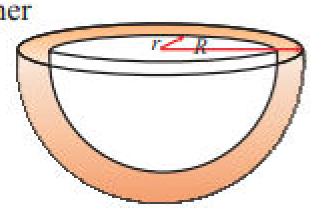


Fig. 8.41

Example 8.17

Find the volume of a sphere-shaped metallic shot-put having diameter of 8.4 cm. (Take $\pi = \frac{22}{7}$)

Solution Let r be radius of the metallic shot-put.

Now, $2r = 8.4$ cm $\implies r = 4.2$ cm

Volume of the shot-put, $V = \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10}$$

Thus, the volume of the shot-put $= 310.464$ cu.cm.

Fig. 8.42

Example 8.18

A cone, a hemisphere and cylinder have equal bases. If the heights of the cone and a cylinder are equal and are same as the common radius, then find the ratio of their respective volumes.

Solution Let r be the common radius of the cone, hemisphere and cylinder.

Let h be the common height of the cone and cylinder.

Given that $r = h$

Let V_1, V_2 and V_3 be the volumes of the cone, hemisphere and cylinder respectively.

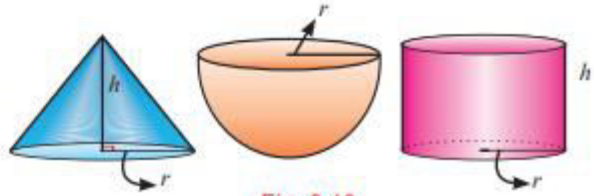


Fig. 8.43

Now, $V_1 : V_2 : V_3 = \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h$

$\Longrightarrow \quad = \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 \qquad$ (here, $r = h$)

$\Longrightarrow V_1 : V_2 : V_3 = \frac{1}{3} : \frac{2}{3} : 1$

Hence, the required ratio is $1 : 2 : 3$.

Example 8.19

If the volume of a solid sphere is $7241\frac{1}{7}$ cu.cm, then find its radius.
(Take $\pi = \frac{22}{7}$)

Solution Let r and V be the radius and volume of the solid sphere respectively.

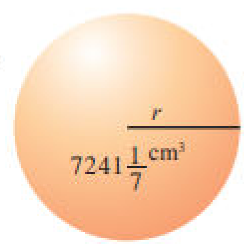


Fig. 8.44

Given that $\quad V = 7241\frac{1}{7}$ cu.cm

$$\Longrightarrow \quad \frac{4}{3}\pi r^3 = \frac{50688}{7}$$

$$\Longrightarrow \quad \frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{50688}{7}$$

$$r^3 = \frac{50688}{7} \times \frac{3 \times 7}{4 \times 22}$$

$$= 1728 = 4^3 \times 3^3$$

Thus, the radius of the sphere, $r = 12$ cm.

Example 8.20

Volume of a hollow sphere is $\frac{11352}{7}$ cm^3. If the outer radius is 8 cm, find the inner radius of the sphere. (Take $\pi = \frac{22}{7}$)

Solution Let R and r be the outer and inner radii of the hollow sphere respectively.

Let V be the volume of the hollow sphere.

Now, given that $V = \frac{11352}{7}$ cm^3

$$\Longrightarrow \quad \frac{4}{3}\pi(R^3 - r^3) = \frac{11352}{7}$$

$$\Longrightarrow \quad \frac{4}{3}\times\frac{22}{7}(8^3 - r^3) = \frac{11352}{7}$$

$$512 - r^3 = 387 \Longrightarrow r^3 = 125 = 5^3$$

Hence, the inner radius, $r = 5$ cm.

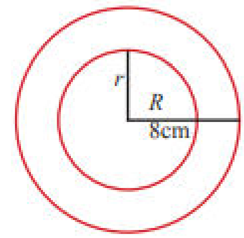


Fig. 8.45

Example 8.21

A solid wooden toy is in the form of a cone surmounted on a hemisphere. If the radii of the hemisphere and the base of the cone are 3.5 cm each and the total height of the toy is 17.5 cm, then find the volume of wood used in the toy. (Take $\pi = \frac{22}{7}$)

Solution Hemispherical portion :

Radius, $r = 3.5$ cm

Conical portion :

Radius, $r = 3.5$ cm

Height, $h = 17.5 - 3.5 = 14$ cm

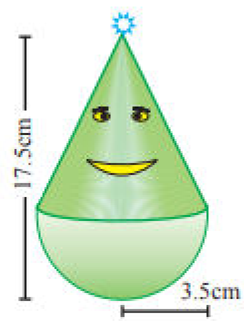


Fig. 8.47

Volume of the wood = Volume of the hemisphere + Volume of the cone

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \frac{\pi r^2}{3}(2r + h)$$

$$= \frac{22}{7}\times\frac{3.5\times 3.5}{3}\times(2\times 3.5 + 14) = 269.5$$

Hence, the volume of the wood used in the toy = 269.5 cu.cm.

Example 8.22

A cup is in the form of a hemisphere surmounted by a cylinder. The height of the cylindrical portion is 8 cm and the total height of the cup is 11.5 cm. Find the total surface area of the cup. (Take $\pi = \frac{22}{7}$)

Solution

Hemispherical portion	Cylindrical portion
Radius, r = Total height – 8	Height, $h = 8$ cm.
$\Longrightarrow r = 11.5 - 8 = 3.5$ cm	Thus, radius $r = 3.5 \text{ cm} = \frac{7}{2}$ cm

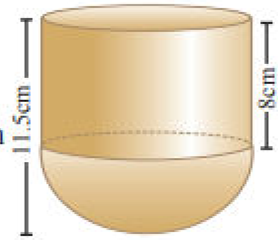


Fig. 8.48

Total surface area of the cup = $\begin{cases} \text{CSA of the hemispherical portion} \\ + \text{ CSA of the cylindrical portion} \end{cases}$

$$= 2\pi r^2 + 2\pi rh = 2\pi r(r+h)$$

$$= 2 \times \frac{22}{7} \times \frac{7}{2}\left(\frac{7}{2} + 8\right)$$

$\therefore$ Total surface area of the cup = 253 sq.cm.

Example 8.23

A circus tent is to be erected in the form of a cone surmounted on a cylinder. The total height of the tent is 49 m. Diameter of the base is 42 m and height of the cylinder is 21 m. Find the cost of canvas needed to make the tent, if the cost of canvas is ₹12.50/m^2. (Take $\pi = \frac{22}{7}$)

Solution

Cylindrical Part	Conical Part
Diameter, $2r = 42$ m	Radius, $r = 21$ m
Radius, $r = 21$ m	Height, $h_1 = 49 - 21 = 28$ m
Height, $h = 21$ m	Slant height, $l = \sqrt{h_1^2 + r^2}$
	$= \sqrt{28^2 + 21^2}$
	$= 7\sqrt{4^2 + 3^2} = 35$ m

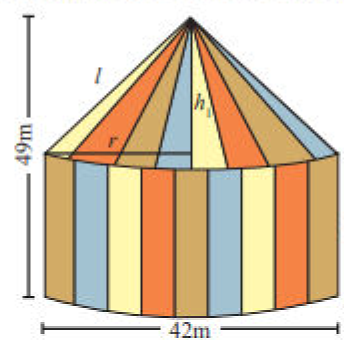


Fig. 8.49

Total area of the canvas needed = CSA of the cylindrical part + CSA of the conical part

$$= 2\pi rh + \pi rl = \pi r(2h + l)$$

$$= \frac{22}{7} \times 21(2 \times 21 + 35) = 5082$$

Therefore, area of the canvas = 5082 m^2

Now, the cost of the canvas per sq.m = ₹12.50

Thus, the total cost of the canvas = 5082 × 12.5 = ₹63525.

Example 8.24

A hollow sphere of external and internal diameters of 8 cm and 4 cm respectively is melted and made into another solid in the shape of a right circular cone of base diameter of 8 cm. Find the height of the cone.

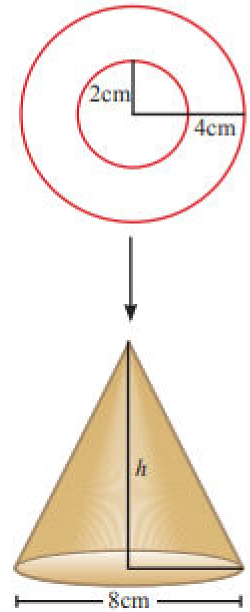


Fig. 8.50

Solution Let R and r be the external and internal radii of the hollow sphere.

Let h and r_1 be the height and the radius of the cone to be made.

Hollow Sphere			
	External	Internal	Cone
	$2R = 8$ cm	$2r = 4$ cm	$2r_1 = 8$
$\Longrightarrow$	$R = 4$ cm	$\Longrightarrow r = 2$ cm	$\Longrightarrow r_1 = 4$

When the hollow sphere is melted and made into a solid cone, we have

Volume of the cone = Volume of the hollow sphere

$$\Longrightarrow \quad \frac{1}{3}\pi r_1^2 h = \frac{4}{3}\pi[R^3 - r^3]$$

$$\Longrightarrow \quad \frac{1}{3}\times\pi\times 4^2\times h = \frac{4}{3}\times\pi\times(4^3 - 2^3)$$

$$\Longrightarrow \quad h = \frac{64-8}{4} = 14$$

Hence, the height of the cone $h = 14$ cm.

Example 8.25

Spherical shaped marbles of diameter 1.4 cm each, are dropped into a cylindrical beaker of diameter 7 cm containing some water. Find the number of marbles that should be dropped into the beaker so that the water level rises by 5.6 cm.

Solution Let n be the number of marbles needed. Let r_1 and r_2 be the radii of the marbles and cylindrical beaker respectively.

Marbles	Cylindrical Beaker
Diameter, $2r_1 = 1.4$ cm	Diameter, $2r_2 = 7$ cm
Radius $r_1 = 0.7$ cm	Radius, $r_2 = \frac{7}{2}$ cm

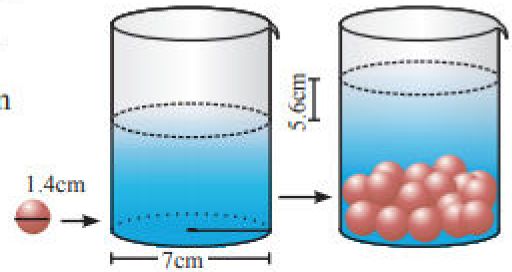


Fig. 8.51

Let h be the height of the water level raised.

Then, $h = 5.6$ cm

After the marbles are dropped into the beaker,

Volume of water raised = Volume of n marbles

$$\Longrightarrow \quad \pi r_2^2 h = n \times \frac{4}{3}\pi r_1^3$$

Thus,
$$n = \frac{3r_2^2 h}{4r_1^3}$$

$$n = \frac{3 \times \frac{7}{2} \times \frac{7}{2} \times 5.6}{4 \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}} = 150.$$

$\therefore$ The number of marbles needed is 150.

Example 8.26

Water is flowing at the rate of 15 km / hr through a cylindrical pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. In how many hours will the water level in the tank raise by 21 cm? (Take $\pi = \frac{22}{7}$)

Solution Speed of water = 15 km / hr

= 15000 m / hr

Diameter of the pipe, $2r = 14$ cm

Thus, $r = \frac{7}{100}$ m.

Let h be the water level to be raised.

Thus, $h = 21 \text{ cm} = \frac{21}{100}$ m

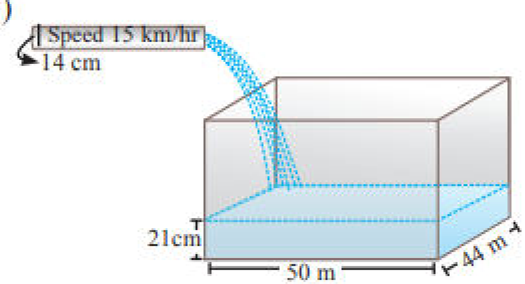


Fig. 8.52

Now, the volume of water discharged

$$= \text{Cross section area of the pipe} \times \text{Time} \times \text{Speed}$$

Volume of water discharged in one hour

$$= \pi r^2 \times 1 \times 15000$$
$$= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000 \text{ cu.m}$$

Volume of required quantity of water in the tank is,

$$lbh = 50 \times 44 \times \frac{21}{100}$$

Assume that T hours are needed to get the required quantity of water.

$\therefore$ $\left.\begin{array}{r}\text{Volume of water discharged} \\ \text{in T hours}\end{array}\right\} = \text{Required quantity of water in the tank}$

$$\Longrightarrow \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times T \times 15000 = 50 \times 44 \times \frac{21}{100}$$

Thus, $T = 2$ hours.

Hence, it will take 2 hours to raise the required water level.

Example 8.27

A cuboid shaped slab of iron whose dimensions are $55\text{ cm} \times 40\text{ cm} \times 15\text{ cm}$ is melted and recast into a pipe. The outer diameter and thickness of the pipe are 8 cm and 1 cm respectively. Find the length of the pipe. (Take $\pi = \frac{22}{7}$)

Solution Let h_1 be the length of the pipe.

Let R and r be the outer and inner radii of the pipe respectively.

Iron slab: Let $lbh = 55 \times 40 \times 15$.

Iron pipe:

Outer diameter, $2R = 8\text{cm}$

$\therefore$ Outer radius, $R = 4\text{ cm}$

Thickness, $w = 1\text{ cm}$

$\therefore$ Inner radius, $r = R - w = 4 - 1 = 3\text{ cm}$

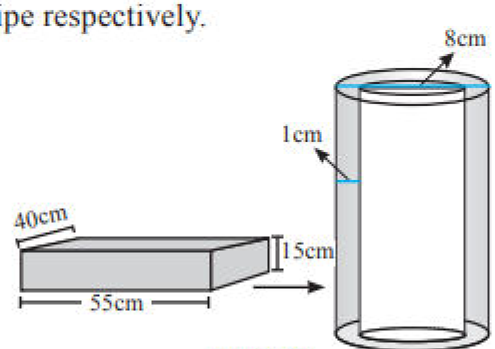


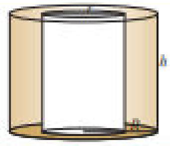
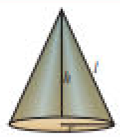
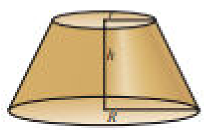
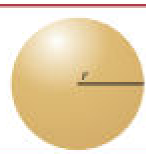
Fig. 8.53

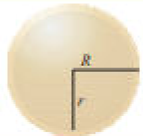
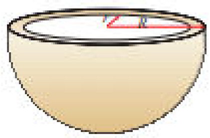
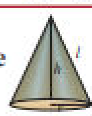
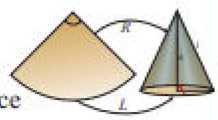
Now, the volume of the iron pipe = Volume of iron slab

$$\Longrightarrow \pi h_1 (R + r)(R - r) = lbh$$

That is, $\frac{22}{7} \times h_1(4 + 3)(4 - 3) = 55 \times 40 \times 15$

Thus, the length of the pipe, $h_1 = 1500\text{ cm} = 15\text{ m}$.

Sl. No	Name	Figure	Lateral or Curved Surface Area (sq.units)	Total Surface Area (sq.units)	Volume (cu.units)
1	**Solid right circular cylinder**		$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$
2	**Right circular hollow cylinder**		$2\pi h(R+r)$	$2\pi(R+r)(R-r+h)$	Volume of the material used $\pi R^2 h - \pi r^2 h$ $= \pi h(R^2 - r^2)$ $= \pi h(R+r)(R-r)$
3	**Solid right circular cone**		πrl	$\pi r(l+r)$	$\frac{1}{3}\pi r^2 h$
4	**Frustum**		- - - - - - -	- - - - - - - - - - - - - - -	$\frac{1}{3}\pi h(R^2 + r^2 + Rr)$
5	**Sphere**		$4\pi r^2$	- - -	$\frac{4}{3}\pi r^3$

5	**Sphere**		$4\pi r^2$	- - -	$\frac{4}{3}\pi r^3$
6	**Hollow sphere**		- - -	- - -	Volume of the material used $\frac{4}{3}\pi(R^3 - r^3)$
7	**Solid Hemisphere**		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
8	**Hollow Hemisphere**		$2\pi(R^2 + r^2)$	$2\pi(R^2 + r^2) + \pi(R^2 - r^2)$ $= \pi(3R^2 + r^2)$	Volume of the material used $\frac{2}{3}\pi(R^3 - r^3)$
9	**A sector of a circle converted into a Cone**  CSA of a cone = Area of the sector $\pi rl = \frac{\theta}{360} \times \pi r^2$ Length of the sector = Base circumference of the cone		$l = \sqrt{h^2 + r^2}$ $h = \sqrt{l^2 - r^2}$ $r = \sqrt{l^2 - h^2}$	10. Volume of water flows out through a pipe = {Cross section area × Speed × Time } 11. No. of new solids obtained by recasting $= \frac{\text{Volume of the solid which is melted}}{\text{volume of one solid which is made}}$		
12	Conversions	1 m³ = 1000 litres , 1 d.m³ = 1 litre , 1000 cm³ = 1 litre , 1000 litres = 1 *kl*			